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Time scale ratios and critical dynamics

An overview is given of recent results concerning systems described by a set of at least two slow dynamical variables. The simplest model contains a relaxing order parameter coupled to the energy density (model C). The effects induced by randomness in such a model are discussed. At the superconducting transition the gauge dependence of the critical dynamics is considered for a model of two coupled relaxation equations.

1. Introduction

Near a phase transition one observes the phenomenon of critical slowing down for the order parameter (OP) dynamics. The time scale for reaching the equilibrium state increases when the critical point is approached. Thus the dynamics separates into slow and fast dynamic variables. A correct description of the critical dynamics has to take into account all slow variables besides the OP. These are the densities of conserved quantities (CD). The dynamic universality classes therefore depend on the structure of the system of these variables, namely on the number of CDs and the type of the coupling to the OP. These dynamic unversality classes have been reviewed by Hohenberg and Halperin.[1]

In principle each of the dynamics variables, which have to be taken into account has its own time scale but near the critical point in many cases it was observed that the time scales of all variables behave in the same way and this was the basis for the dynamic scaling hypothesis which characterized the critical dynamics by *one* dynamical critical exponent z defined by the dispersion $\omega_c(k) = Ak^z$ of the OP characteristic frequency at the critical temperature T_c, where k is the wave vector modulus and A a non universal amplitude setting a time scale. Later on it was recognized that there are cases where the critical dynamics cannot be described by only one critical time scale but the time scales of the OP and the CDs might be different.

We shall review here some examples where this is the case.

2. The time scale ratio in a simple dynamic model

The simplest example where time scale ratios (TSRs) can be studied is a system whose critical dynamics is defined by a nonconserved OP $\vec{\phi}_0$, described by a relaxation equation, coupled to one CD m_0, described by a diffusive equation.[2] The coupling of the OP and the CD is accomplished by a term in the static functional H, which enters the restoring force. This model was named[1] model C and reads explicitly

$$\frac{\partial \vec{\phi}_0}{\partial t} = -\overset{o}{\Gamma}\frac{\delta H}{\delta \vec{\phi}_0} + \vec{\theta}_\phi \qquad \frac{\partial m_0}{\partial t} = \overset{o}{\lambda}\nabla^2\frac{\delta H}{\delta m_0} + \theta_m \tag{1}$$

The stochastic forces fulfill Einstein relations which assure an approach of an equilibrium described by the static functional

$$H = \int d^d x \left\{ \frac{1}{2}\overset{o}{\tau}(\vec{\phi}_0 \cdot \vec{\phi}_0) + \frac{1}{2}\sum_{i=1}^{n} \vec{\nabla}\phi_{i0}\cdot\vec{\nabla}\phi_{i0} + \frac{\overset{o}{\tilde{u}}}{4!}(\vec{\phi}_0 \cdot \vec{\phi}_0)^2 \right.$$
$$\left. + \frac{1}{2}a_m m_0^2 + \frac{1}{2}\overset{o}{\gamma}\, m_0(\vec{\phi}_0 \cdot \vec{\phi}_0) - \overset{o}{h}_m\, m_0 \right\}. \tag{2}$$

The usual ϕ^4 theory has been extended by a Gaussian part for the CD and the asymmetric coupling $\overset{o}{\gamma}$ between the CD and the OP squared. The important dynamical parameter is the TSR $\overset{o}{w} = \overset{o}{\Gamma} / \overset{o}{\lambda}$

We applied the field theoretic formalism[3] to this model and calculated the fixed point (FP) value of the TSR as function of space dimension d ($\epsilon = 4 - d$) and number of components n of the OP (the so called 'phase diagram') in two loop order.[5,6] It turns out that the FP value of the TSR might be (i) nonzero and finite, (ii) zero or (iii) infinite. Case (i) is the so called *strong scaling* FP, case (ii) the *weak scaling* FP and (iii) a FP with unclear scaling properties.[2,4] In the region of the (ϵ, n)-space where the specific heat does not diverge the FP value of the asymmetric coupling $\overset{o}{\gamma}$ is zero and the two equations decouple. Then the system belongs to the universality class of a simple relaxational model (model A). The CD then may be characterized by a dynamic exponent $z_{CD} = 2$. On the basis of the correct two loop field theoretic functions[5,6] one concludes that the FP of case (iii) does not exist (see Fig. 1 (a)). An infinite FP value of the TSR w (quantities without a super or subscript zero are renormalized)is suppressed by a logarithmic term $\ln w$ in the ζ-function for the relaxation

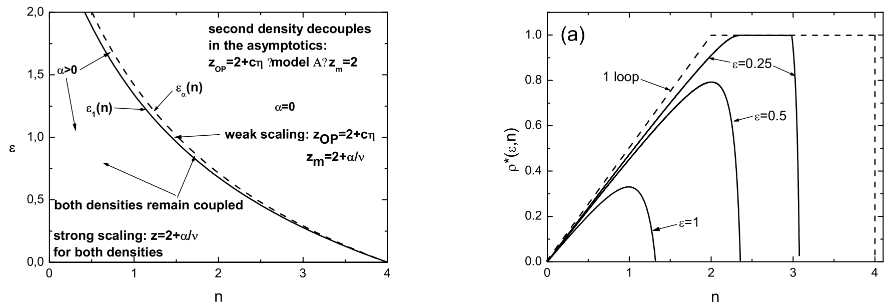

Fig. 1. (a) 'Phase diagram of model C'; (b) Time scale ratio of model C ($\rho^\star = w^\star/(1+w\star)$).

rate Γ, however such a term leads to a nonanalytic dependence of the TSR in the limit where ϵ goes to zero ($d \to 4$).

3. Randomness and the time scale ratio

One may ask how defects and randomness influence the dynamic critical behavior of model C. Randomness can be induced by several effects eg. (i) bond disorder, (ii) site disorder or (iii) anisoropic axis disorder. This is shown in the following spin-Hamiltonian

$$\mathcal{H} = -\frac{1}{2}\sum_{\mathbf{R},\mathbf{R}'} J(|\mathbf{R}-\mathbf{R}'|)c_{\mathbf{R}}c_{\mathbf{R}'}\vec{S}_{\mathbf{R}}\vec{S}_{\mathbf{R}'} - D_0\sum_{\mathbf{R}}(\hat{x}_{\mathbf{R}}\vec{S}_{\mathbf{R}})^2,$$

where the disorder is defined in case (i) by a distribution of the spin couplings J like $p(J) = \exp\left(-J^2/\Delta\right)$, in case (ii) by probability $p(c)$ of occupation $c = 1$ or vacancy $c = 0$ and in case (iii) by a non-isotropic distribution of the directions of the anisotropy axis $\hat{x}$ $p(\hat{x}) = \frac{1}{2m}\sum_{i=1}^{m}\left[\delta^{(m)}(\hat{x}-\hat{k}_i)+\delta^{(m)}(\hat{x}+\hat{k}_i)\right]$.

From static considerations formulated in the so called Harris criterion[7] one may argue in the following: If the pure system's *specific heat is diverging* then the critical exponents may be changed by disorder (this is the case for the examples given). The disordered system is then characterized by a non-diverging specific heat. Otherwise disorder remains in the universality class of the pure system.If the *specific heat is not diverging* the coupling γ of a CD to the OP goes to zero and model A applies (see Fig 1(a) left to the dashed line). Therefore the coupling of a conserved density is in any case irrelevant. However this is only an argument which holds in the asymptotics. From statics one knows[8,9] that the static critical behavior observed might

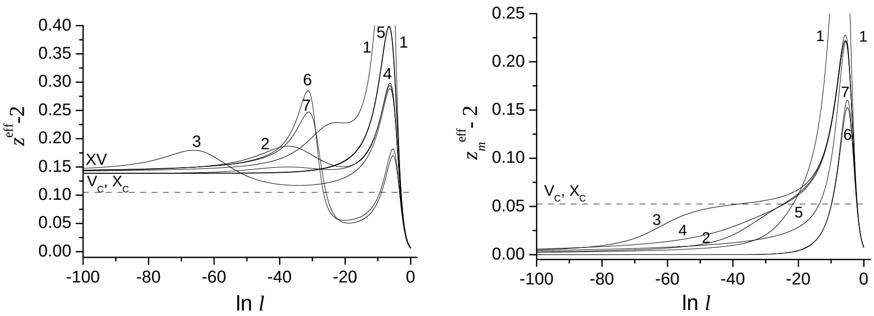

Fig. 2. Effective dynamical critical exponents in model C for a Heisenberg magnet with random anisotropy (from[11]) (a) for the OP (b) for the CD.

be an *effective* one, therefore one may consider also effective dynamical critical behavior.[10,11] Therefore *effective* dynamical critical exponents are defined by the field-theoretic function ζ_Γ of the kinetic coefficient Γ of the OP and ζ_m of the CD m

$$z^{\rm eff} = 2 + \zeta_\Gamma(\{u_i(\ell)\}, \gamma(\ell), w(\ell)), \qquad z_m^{\rm eff} = 2 + \zeta_m(\{u_i(\ell)\}, \gamma^2(\ell)). \quad (3)$$

They depend on the solution of the flow equations of the static model parameters $u_i(\ell)$, $\gamma(\ell)$, and the TSR $w(\ell)$. It turns out that including a CD leads to a new *small* dynamic transient exponent.[12] Thus nonasymptotic effects might be observable. In such a case the effective scaling of the OP and CD are in general different as shown in Fig. 2. Quite recently the asymptotic critical dynamics of model A for the Ising model has been studied by computer simulations[13] and it has been demonstrated that (as one expected) the dynamics of case (i) and (ii) belong to the same universality class.

4. Gauge dependence of the time scale ratio

The static critical behavior of a superconductor is described by a complex OP $\vec{\psi}$ (generalized to $n/2$ components) and the gauge field $\vec{A}$ coupled to the OP by the minimal coupling. The corresponding static functional reads

$$\mathcal{H} = \int d^d x \Big\{ \frac{1}{2}\mathring{r}|\vec{\psi}_0|^2 + \frac{1}{2}\sum_{i=1}^{n/2} |(\boldsymbol{\nabla} - i\mathring{e}\boldsymbol{A}_0)\psi_{0,i}|^2 + \frac{\mathring{u}}{4!}(|\vec{\psi}_0|^2)^2 + \frac{1}{2}(\boldsymbol{\nabla}\times\boldsymbol{A}_0)^2 + \frac{1}{2\mathring{\varsigma}}(\boldsymbol{\nabla}\cdot\boldsymbol{A}_0)\Big\}$$

This coupling is due to the charge of the electrons building the condensating Cooper pairs. Renormalization group theory calculated a 'charged' FP describing the critical behavior of superconductors of the second kind. Experimental verification has been found by measuring the behavior of the penetration depth.[14] Moreover RG theory predicts that nonmeasurable quantities may show a dependence on the gauge choosen in the calculation, whereas measurable quantities have to be gauge independent. Thus some of the static critical exponents like γ (OP susceptibility) or η (decay at T_c of the OP correlations) are gauge dependent whereas exponents like ν or α are gauge independent. The usual scaling laws are completely consistent with this behavior.[15]

Recently a dynamical model has been suggested for the critical dynamics of superconductors of the second kind[16] consisting of two coupled relaxational equations for the OP and the vector potential as follows

$$\frac{\partial\psi_{0,i}}{\partial t} = -2\mathring{\Gamma}_\psi \frac{\delta\mathcal{H}}{\delta\psi^+_{0,i}} + \theta_i\,, \qquad \frac{\partial A_{0,\alpha}}{\partial t} = -\mathring{\Gamma}_A \frac{\delta\mathcal{H}}{\delta A_{0,\alpha}} + \theta_\alpha.$$

The TSR is now defined as $w = \Gamma/\Gamma_A$. At the weak scaling FP (here $w^\star = \infty$) different time scales for the characteristic frequencies are obtained

$$\omega_\psi \sim k^{z_\psi} g_\psi(k\xi) \quad \omega_A \sim k^{z_A} g_A(k\xi) \tag{4}$$

and z_A is found to be gauge independent as it must be since it can be measured via the frequency dependent conductivity[17] $\sigma(k = 0, \omega) = \xi^{z_A-2+\eta_A}\mathcal{G}(\omega\xi^{z_A}) \sim \xi^{z_A+2-d}$ in the limit $\xi \to \infty$ with the exact result for the static exponent $\eta_A = 4 - d$ and a finite value of the scaling function $\mathcal{G}(\infty)$. But this FP is dynamically unstable. At the stable strong scaling FP ($w^\star$ finite and gauge dependent) $z = z_\psi = z_A = 2 + \frac{18}{n}\varepsilon - \varsigma\frac{6}{n}\frac{\varepsilon}{1+w^\star}$ thus z_A turns out to be gauge dependent.[18] In consequence the strong scaling FP of this model cannot describe the critical dynamics. Either the model does not apply or the stability of the FPs is changed in higher loop order.

5. Outlook

Two other longstanding problems (i) the dynamical critical behavior at the tricritical point in ^{3}He-^{4}He mixtures[21] and (ii) the dynamical critical scattering above the Neel temperature T_N of the three-dimensional Heisenberg antiferromagnet[22] are related to the FP value of the TSR w. In both cases the critical dynamics is described by a model more complicated than model C, containing mode coupling terms in the dynamic equations. In (i) the FP value of one of the TSRs turned out to be infinite in a one loop calculation[19] (similar to model C). In two loop order (for the same reason as in model C) this FP is absent and it turns out that the mass diffusion is diverging contrary to measurement[20] (for more details see Ref.[21]). In the second case the FP value of the TSR between the kinetic coefficients of the staggered magnetization and the magnetization changes in two loop order[22] from roughly $w^\star = 3$ to $w^\star = 1$, which changes the dynamic shape function. This has to be taken into account in the comparison with experiment[23] since it changes the size of the additional elastic component observed in the critical scattering.[24]

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