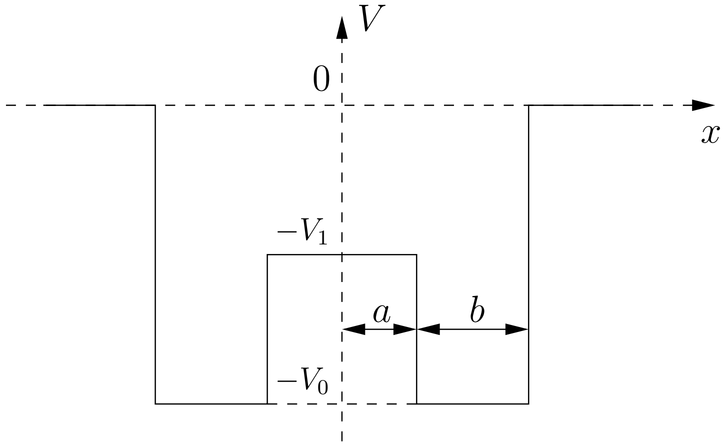
9. A double quantum well is described by the potential given in the figure below.

   (a) Use the symmetry of the potential to simplifiy the problem and derive an equation for the energy eigenvalues of the bound states of the problem.

   (b) Describe the strategy you would use to solve the problem numerically (*i.e.*: which equation to you have to solve for what variable, how do you calculate the rest of the unknowns?)

   (c) (voluntary) Solve this equation numerically for $a = 1$, $b = 2$, $V_0 = \frac{\hbar^2}{2m}$, $V_1 = 0$.

   (d) (voluntary) For the Mathematica-savvy persons among you: Try to make plots of the wave functions for the first few bound states. Can you recognize some pattern? Compare them to the bound states of a single quantum well!



10. Consider a particle moving in an infinitely deep potential well.

    (a) Calculate $\langle x^2 \rangle - \langle x \rangle^2$ and $\langle p^2 \rangle - \langle p \rangle^2$ and show that the uncertainty principle is fulfilled.

    (b) Calculate $\langle x^2 \rangle - \langle x \rangle^2$ for a classical particle moving in the same potential well and show that the result agrees with the quantum mechanical one for large quantum numbers.

11. A,B and C are abritrary quantum-mechanical operators.

    (a) Show that: $[AB, C] = A[B, C] + [A, C]B$

    (b) Calculate: $[\mathbf{p}^2, \mathbf{L}]$ where $\mathbf{p} = -i\hbar\nabla$ and $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

    (c) Calculate: $[\frac{1}{r}, \mathbf{p} \times \mathbf{L}]$

12. Consider a particle with mass m in an one-dimensional potential

$$V(x) = \begin{cases} \infty, & x < 0 \\ -V_0, & 0 \leq x \leq a \\ 0, & x > a \end{cases} .$$

Show that the wave function $\psi(x)$ for a bound state can be continued to be an odd wave function belonging to a bound state of a rectangular potential well with width $2a$ and depth V_0. What is the number of bound states in dependence of a and V_0? What energy do they have? Does a bound state always exist?