17. Functions of operators are defined by their Taylor series. One important function is the exponential function, as the time evolution of a quantum system described by a time-independent Hamiltonian H is given by

$$|\psi(\mathbf{r},t)\rangle = e^{-\frac{i}{\hbar}Ht}|\psi(\mathbf{r},0)\rangle.$$

Asssuming that you already know the eigenfunctions $\varphi_i(\mathbf{r})$ and corresponding eigenvalues E_i of the Hamiltonian H, calculate the time evolution of $|\psi(\mathbf{r},t)\rangle$ by using the spectral representation of H to evaluate the Taylor series.

18. Let A, B and C be pairwise *non*commuting operators describing measurements in a quantum-mechanical system. You know that a measurement of A at time t_0 gave the result a_0. Find the probability that a measurement of C at a later time gives the result C_3, if

   (a) a measurement of B immediately after measuring A yielded b_1, and the measurement of C is carried out immediately after B.

   (b) B was measured immediately after A, and C is measured immediately after B, but you don't know the result of B.

   (c) C was measured immediately after A.

19. You are given the following matrices A and B defined in $\mathbb{R}^3 \times \mathbb{R}^3$:

$$A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & 2 & -1 \\ 2 & 2 & -1 \\ -1 & -1 & 5 \end{pmatrix}.$$

Under which condition can you find a common set of eigenvectors for A and B? Show that this condition is fulfilled for A and B given above. Use the eigensolutions of A to calculate the eigenvectors and eigenvalues of B in a subspace. In which basis are A and B simultanously diagonal?

20. In a two-dimensional complex Hilbert-space a linear operator is defined by his action on the vectors of an orthonormal basis $\{|e_1\rangle, |e_2\rangle\}$:

$$A|e_1\rangle := -|e_2\rangle, \qquad A|e_2\rangle := |e_1\rangle$$

   (a) Constitute the operator A as linear combination of ket-bra-expressions $|e_j\rangle\langle e_k|, j, k = 1, 2$.

   (b) Is A a normal-operator? Is A self-adjoint? Is A unitary? Is A idempotent?

   (c) Does A only exhibit real Eigenvalues? Does A exhibit orthogonal Eigenvectors?

Hint: A bounded operator is called a normal operator, if it commutes with its self-adjoint. Normal operators possess a complete system of orthogonal Eigenvectors.