25. In class, you have calculated the commutators $[L_i, L_k]$, $[x_i, L_k]$, $[p_i, L_k]$ by studying the behaviour of the operators under infinitesimal rotations.

    Use analogous arguments for infinitesimal translations to calculate $[p_i, x_k]$.

    Hint: to determine the translation operator, examine the commutator of two translation operators $T(a_1)$ and $T(a_2)$. What does the composition of the operators do?

26. (a) Find the matrix representations of the operators L^2, L_x, L_y, L_z, L_+ and L_- in the basis of the eigenstates of L^2 and L_z for the case $\ell = 2$ (ℓ is the quantum number belonging to L^2).

    (b) Calculate the action of L_+ and L_- on the basis functions.

    (c) Determine the real space representation of the basis states. Start with $L_+ Y_\ell^\ell = 0$ and use the relation $L_- Y_\ell^\ell \propto Y_\ell^{\ell-1}$. Correctly normalize the states!

27. At a particular time a particle with mass m is described by state function (one-dimensional Problem; N normalization constant)

$$\psi(x) = \begin{cases} Nxe^{-ax} & \text{für} \quad x \geq 0 \\ 0 & \text{für} \quad x < 0 \end{cases}, \qquad a > 0$$

    What is the probability at the particular time that the measurement of the momentum gives a value between $-\hbar a$ and $+\hbar a$? *Hint:* Use the formulas

$$\int \frac{d\xi}{(\xi^2 + \alpha^2)^2} = \frac{\xi}{2\alpha^2(\xi^2 + \alpha^2)} + \frac{1}{2\alpha^3} \arctan \frac{\xi}{\alpha} + C, \qquad \alpha \in \mathbb{R}^+$$

$$\int_0^\infty d\xi \xi^n e^{-\xi} = n!, \qquad n \in \mathbb{N}_0$$

28. Show: The state function of a particle in space representation is real-valued, it is essential

    (a) The probability density concerning the momentum in momentum space is inversion-symmetric relative to the origin.

    (b) The expectation value of the particle momentum is zero.