

37. A particle of mass  $m$  and charge  $q$  moves in a magnetic field  $\mathbf{B} = B \mathbf{e}_z$ . Determine the energy eigenvalues and the corresponding eigenvectors.

*Hint:* Express the Hamiltonian using the conjugate momenta  $\Pi_x$  and  $\Pi_y$ . Determine the commutator relations of these momentum operators. Now introduce ladder operators like you did for the harmonic oscillator.

38. A spin  $s = \frac{3}{2}$  is determined by a state  $|\chi\rangle$ , which possesses the expectation value  $+\frac{\hbar}{2}$  for the z-component of the spin.
- Can we conclude, that the vector  $|\chi\rangle$  has to be the eigenvector of  $S_z$ ?
  - Will the conclusion of (a.) be changed, if we assume additionally, that expectation values of  $S_x$  and  $S_y$  are zero in the state  $|\chi\rangle$ ?

39. A electron is located in the magnetic field  $\vec{B}$ , which is given by his vector potential  $\vec{A}$ .
- How does the time-dependent Schrödinger equation look like for the 2-component state function  $\hat{\psi}(\vec{r}, t)$  of the electron in the  $\{\vec{r}m_s\}$ -representation?
  - Show for homogenous magnetic fields, that the dynamics of the spin can be separated from the dynamics of the 'path movement' by the ansatz

$$\hat{\psi}(\vec{r}, t) = \begin{bmatrix} \psi(\vec{r}+, t) \\ \psi(\vec{r}-, t) \end{bmatrix} = \phi(\vec{r}, t)\hat{\chi}(t), \quad \hat{\chi}(t) \begin{bmatrix} \chi(+, t) \\ \chi(-, t) \end{bmatrix}$$

Describe the equations of motion of the state function  $\phi(\vec{r}, t)$  and the spinor  $\hat{\chi}(t)$ .

40. Consider a system of two spins with  $S_1 = S_2 = \frac{1}{2}$ . The state space in the  $|S_{1z}, S_{2z}\rangle$ -representation is  $\{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$ . In the  $|S, S_z\rangle$ -representation (total spin  $\vec{S} = \vec{S}_1 + \vec{S}_2$ ) there are three triplett-state with  $S = 1$  and one singulett-state with  $S = 0$ . What will arise for the singulett- and triplett-state with  $S_z = 0$  if we change to the  $|S_{1x}, S_{2x}\rangle$ -representation?