11^{th} exercise quantum mechanics Institut für theoretische Physik, JKU Linz
SS 2007 Date: 22.06.2007

39. *Mind that problem no. 39 from the previous exercise has been delayed to this exercise.*

41. The magnetic moment of an electron is measured by the operator

$$\vec{M} = \frac{e}{2mc}(\vec{L} + 2\vec{S}).$$

The expectation value of M_z in the state $|j, j_z\rangle$ can be written in the form

$$\langle M_z \rangle = \frac{e\hbar}{2mc} g_L j_z,$$

where g_L is called the Landé factor. Calculate g_L of the states $|\frac{5}{2}, \frac{3}{2}\rangle$ and $|\frac{3}{2}, \frac{3}{2}\rangle$.

42. The Hamiltonian of a system of two spins $s = \frac{1}{2}$ is given by

$$H_S = \frac{a}{\hbar}\left(S_z^{(1)} + S_z^{(2)}\right) + \frac{4b}{\hbar^2}\mathbf{S}^{(1)}\mathbf{S}^{(2)},$$

where $a, b > 0$ and $\mathbf{S}^{(j)}$ is the spin operator of spin number j. Calculate the eigenvalues of H_S and express the corresponding eigenvectors in the basis of the eigenstates

$$\left|\frac{1}{2}\frac{1}{2}m_1 m_1\right\rangle =: |m_1 m_2\rangle = |m_1\rangle^{(1)} |m_2\rangle^{(2)}$$

of $(\mathbf{S}^{(1)})^2$, $(\mathbf{S}^{(2)})^2$, $S_z^{(1)}$ and $S_z^{(2)}$. For which relation a/b are the energies degenerate?

43. (a) Express the eigenvectors of the components S_x and S_y of the spin operator $\mathbf{S}$ of a spin $s = \frac{1}{2}$ in the basis of the eigenstates of S_z.

(b) A spin $s = \frac{1}{2}$ is in the state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1+i}{2}|-\rangle$$

at a certain time. Find the direction in space for which the projection of the spin has zero uncertainity.

Hint: Use the fact that the uncertainity of an operator A in a state $|\Psi\rangle$ is zero, if and only if $|\Psi\rangle$ is an eigenvector of A. Decompose the projection of the spin $\mathbf{s}$ onto an arbitrary direction $\mathbf{e}$ into its components, then replace σ_x and σ_y by operators more suitable to your problem ...