44. A particle with spin $s = \frac{1}{2}$ is moving in the potential

$$V(x) = \begin{cases} V_0\,\sigma_z & x > 0 \\ 0 & x \leq 0 \end{cases}.$$

A monochromatic, polarized stream of particles is coming from $-\infty$ and moving in the direction of the positive x-axis. All particles are in the eigenstate of σ_x corresponding to the eigenvalue 1.

(a) What is the general expression for the wave function of those particles?

(b) Calculate the general solution for the transmitted and reflected waves.

(c) What are the border conditions at $x = 0$? Give a general expression for the probability amplitudes of parts a) and b).

(d) When is the reflected wave an eigenstate of σ_x? When is the transmitted wave an eigenstate of σ_x?

45. Repeat the algebraic treatment of the quantum mechanical harmonic oscillator

$$H = \frac{1}{2m}P^2 + \frac{1}{2}m\omega^2 X^2.$$

(a) Calculate the commutator $[P, X]$. What is $[P, H]$ and $[X, H]$?

(b) Introduce the ladder operators $a^\dagger$, a. What is their commutator? Calculate $[a, H]$ and $[a^\dagger, H]$!

(c) If $|\lambda\rangle$ is an eigenstate of H, what can you conclude for $a|\lambda\rangle$ and $a^\dagger|\lambda\rangle$?

(d) Express the Hamiltonian in terms of the ladder operators.

(e) What are the energy eigenvalues of the harmonic oscillator?

(f) Calculate the action of the ladder operators onto a state $|\lambda\rangle$. (Up to now, you know that $a|\lambda\rangle = c_\lambda|\lambda - \hbar\omega\rangle$ and $a^\dagger|\lambda\rangle = c'_\lambda|\lambda + \hbar\omega\rangle$ — *i.e.* you still have to calculate the constants c_λ and c'_λ).

(g) How can you generate an arbitrary state out of the "vacuum state" $|0\rangle$?

(h) Interpret your results! Why is the form of the Hamiltonian you derived in part (d) called the "particle number representation"?

46. A particle is in the $j = 1$ state of the angular momentum operator $\mathbf{J}^2$. Calculate the matrix representations of the operators J_z, J_+, J_-, J_x, J_y and $\mathbf{J}^2$ in the basis $|1, j_m\rangle$ of the eigenstates of $\mathbf{J}^2$, J_z.

47. A particle moves in the spherically symmetric potential

$$V(r) = -\frac{\hbar^2}{2m}\Omega\delta(r - R).$$

(a) Derive an equation that determines the bound states.

(b) Solve the equation for $l = 0$. Under what condition does at least one bound state exist?