Spin drag in cold Fermi gases

Marco Polini

NEST & SNS, Pisa (Italy)

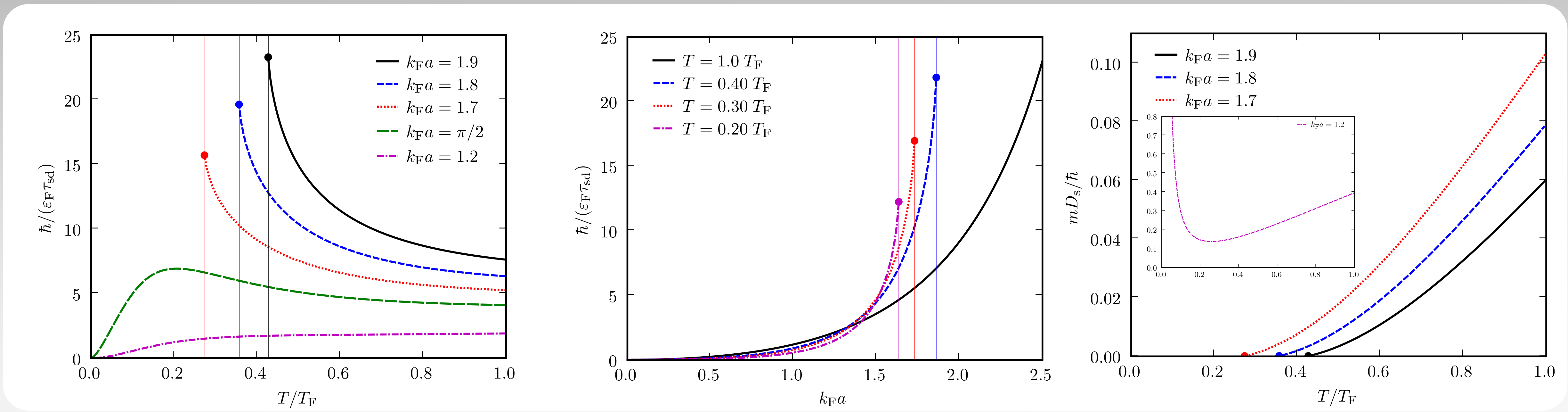
Cosimo I de' Medici
1519-1574

SCUOLA NORMALE SUPERIORE PISA

"Cold gases meet many-body theory", Grenoble (France), August 7th 2010

Collaborators

Rembert Duine (Utrecht, The Netherlands)
Henk Stoof (Utrecht, The Netherlands)
Giovanni Vignale (UMO, USA)



This talk mainly based on:
R.A. Duine, MP, H.T.C. Stoof, and G. Vignale, Phys. Rev. Lett. **104**, 220403 (2010)

Outline

Introduction and motivations

- Coulomb drag between closely spaced electronic circuits
- Coulomb drag close to exciton condensation
- Friction in spin-polarized transport: spin drag
- Itinerant ferromagnetism in a Fermi gas of ultracold atoms

Theory of spin drag in the vicinity of itinerant ferromagnetism

- Model Hamiltonian and Stoner mean-field theory
- Spin-drag relaxation rate
- Effective interactions: density, longitudinal and transverse spin fluctuations

Numerical results

- Spin-drag relaxation rate
- Spin diffusion constant

Conclusions and future perspectives

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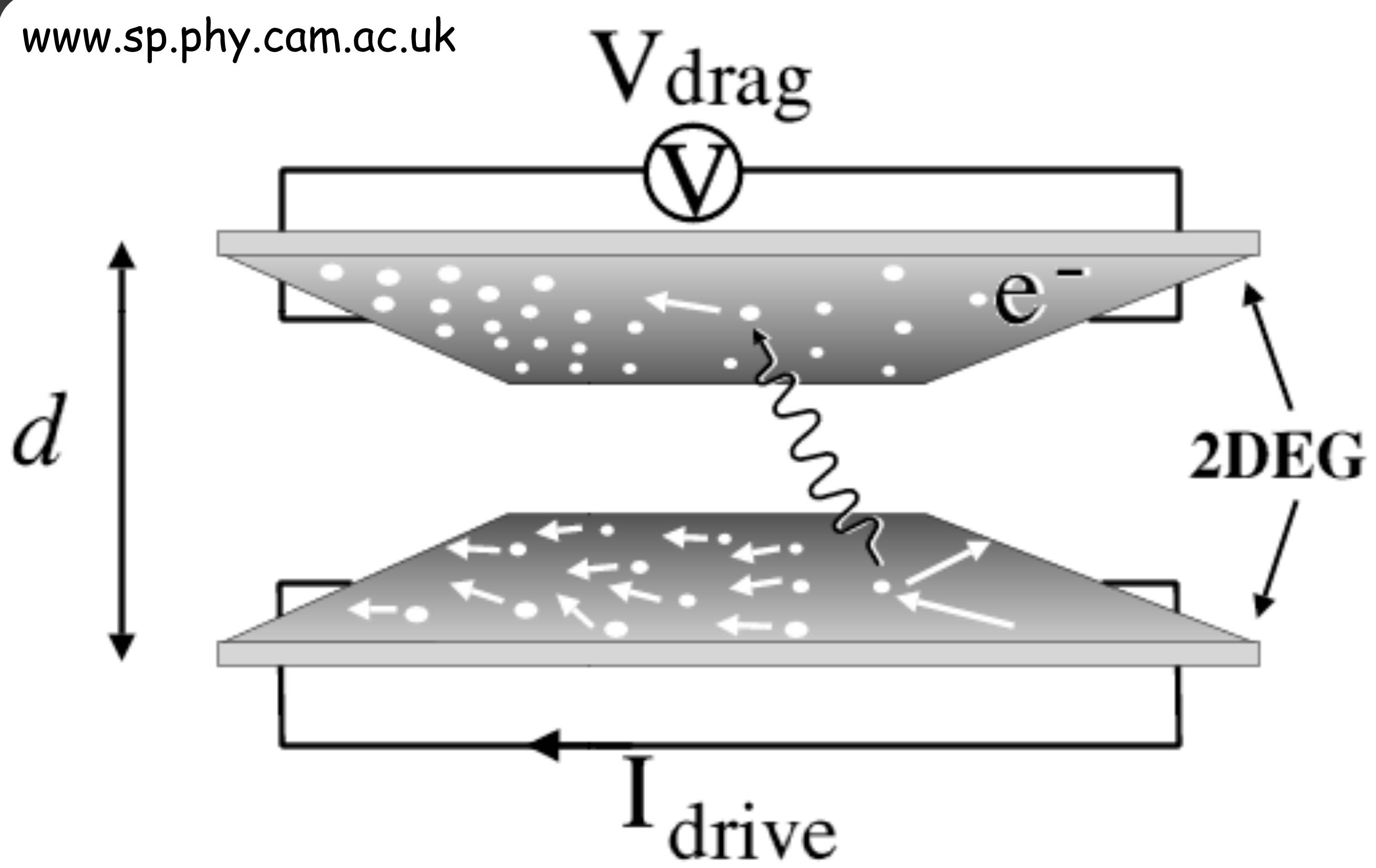
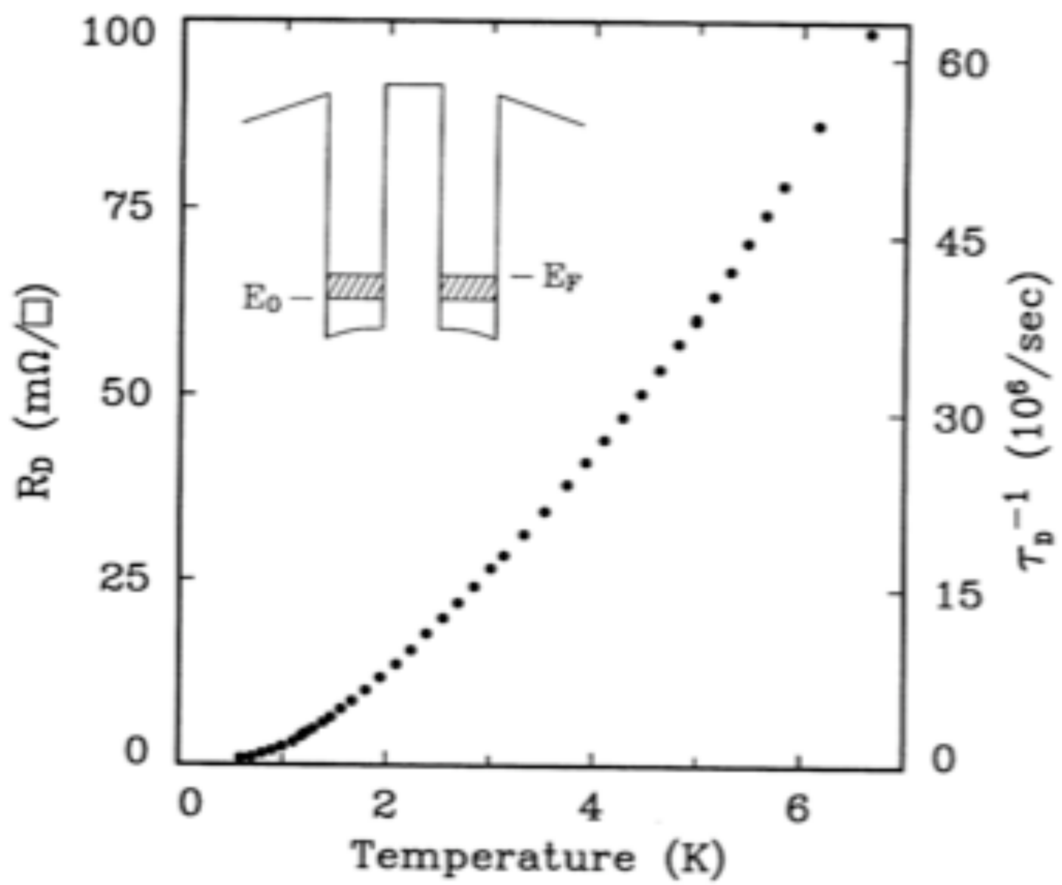
Coulomb drag





M.B. Pogrebinskii, Sov. Phys. Semicond. **11**, 372 (1977)
P.J. Price, Physica **117B**, 750 (1983)
L. Zheng and A.H. MacDonald, Phys. Rev. B **48**, 8203 (1993)
A.-P. Jauho and H. Smith, Phys. Rev. B **47**, 4420 (1993)
...

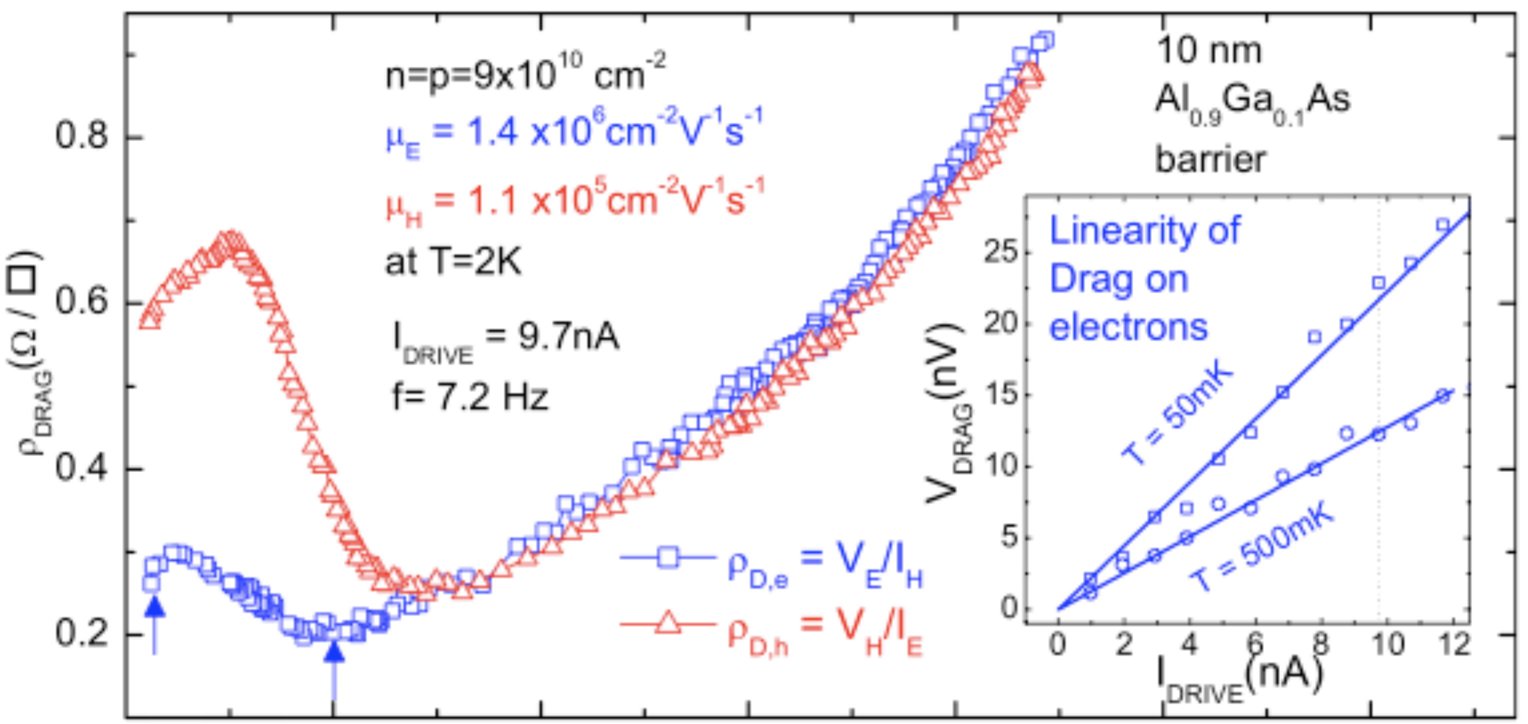
T.J. Gramila et al., Phys. Rev. Lett. **66**, 1216 (1991)

In a Fermi liquid:

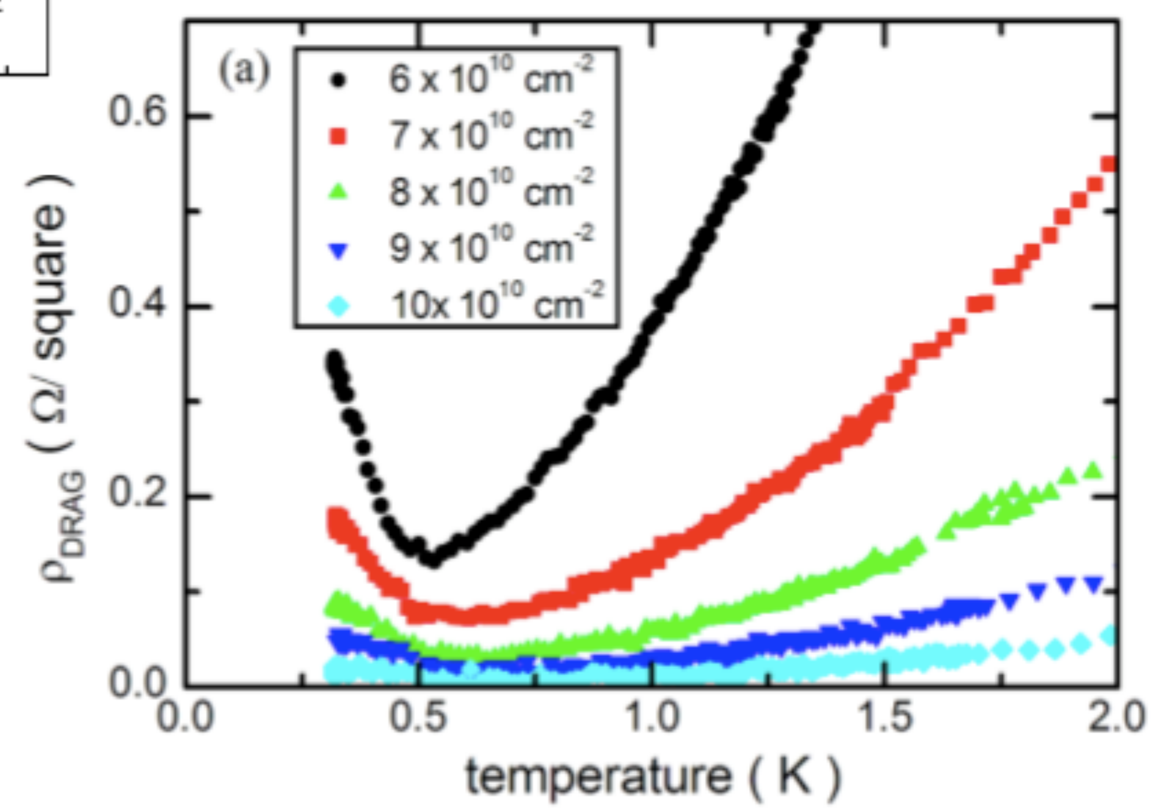
$$R_{\mathrm{D}} \equiv \frac{V_{\mathrm{drag}}}{I_{\mathrm{drive}}} \propto \frac{1}{\tau_{\mathrm{D}}} \sim T^2$$

... what else ?

Coulomb drag in an electron-hole bilayer



A.F. Croxall et al., Phys. Rev. Lett. **101**, 246801 (2008)



J.A. Seamons et al., Phys. Rev. Lett. **102**, 026804 (2009)

Note the anomalous upturn at low T
(signature of exciton condensation ?)

For a theoretical discussion of drag in an e-h bilayer see:
G. Vignale and A.H. MacDonald, Phys. Rev. Lett. **76**, 2786 (1996)
B.Y-.K. Hu, Phys. Rev. Lett. **85**, 820 (2000)

Friction in spin-polarized transport: spin Coulomb drag

Force between particles (electrons, atoms, etc) with antiparallel (pseudo)spin

$$F_{\sigma\bar{\sigma}} = -m \frac{n_{\bar{\sigma}}}{n} \frac{v_{\sigma} - v_{\bar{\sigma}}}{\tau_{\rm sd}}$$

Rate of change of spin-up momentum

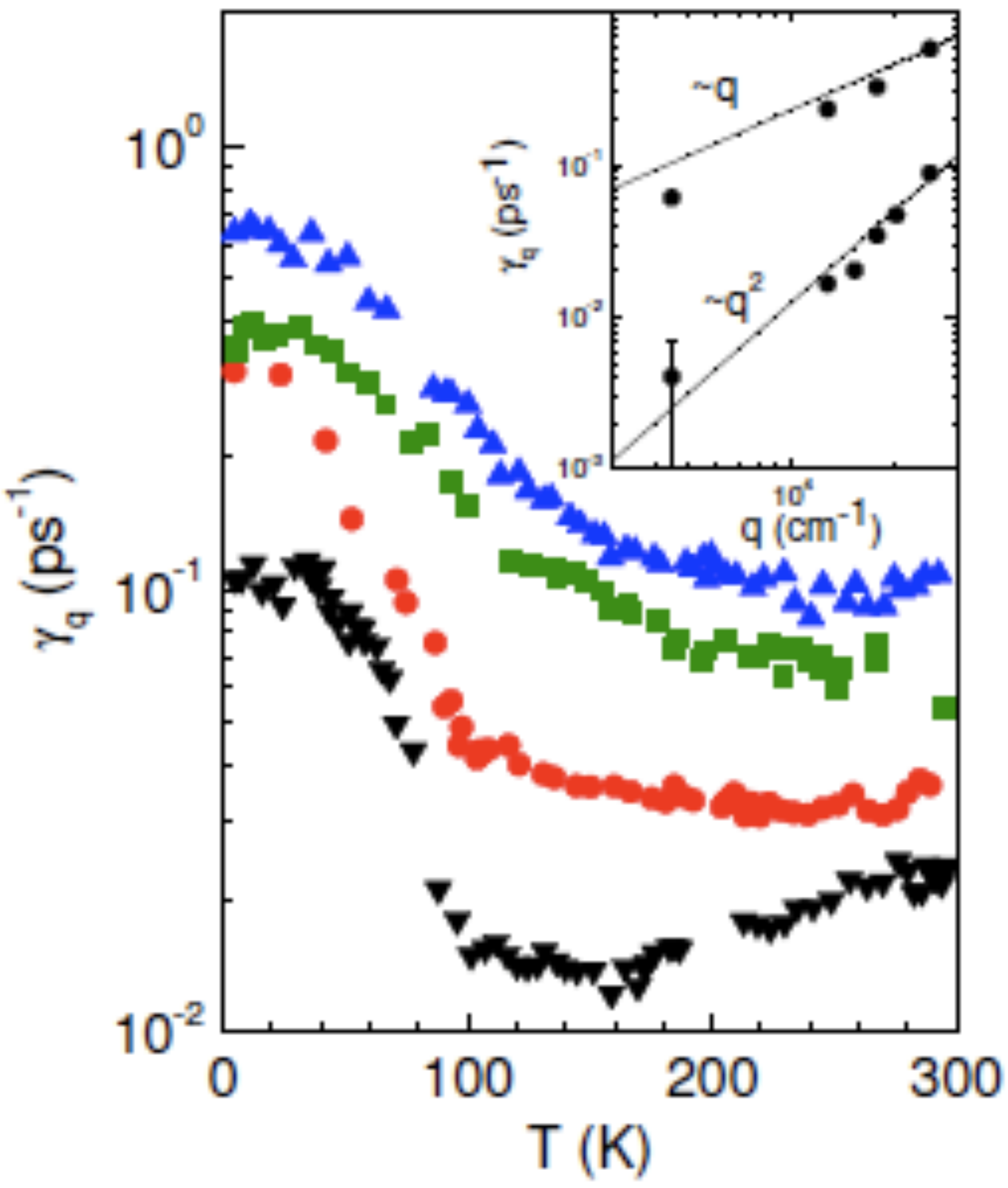
$$\frac{dP_{\uparrow}}{dt} = -\frac{1}{\tau_{\rm sd}} P_{\uparrow}$$

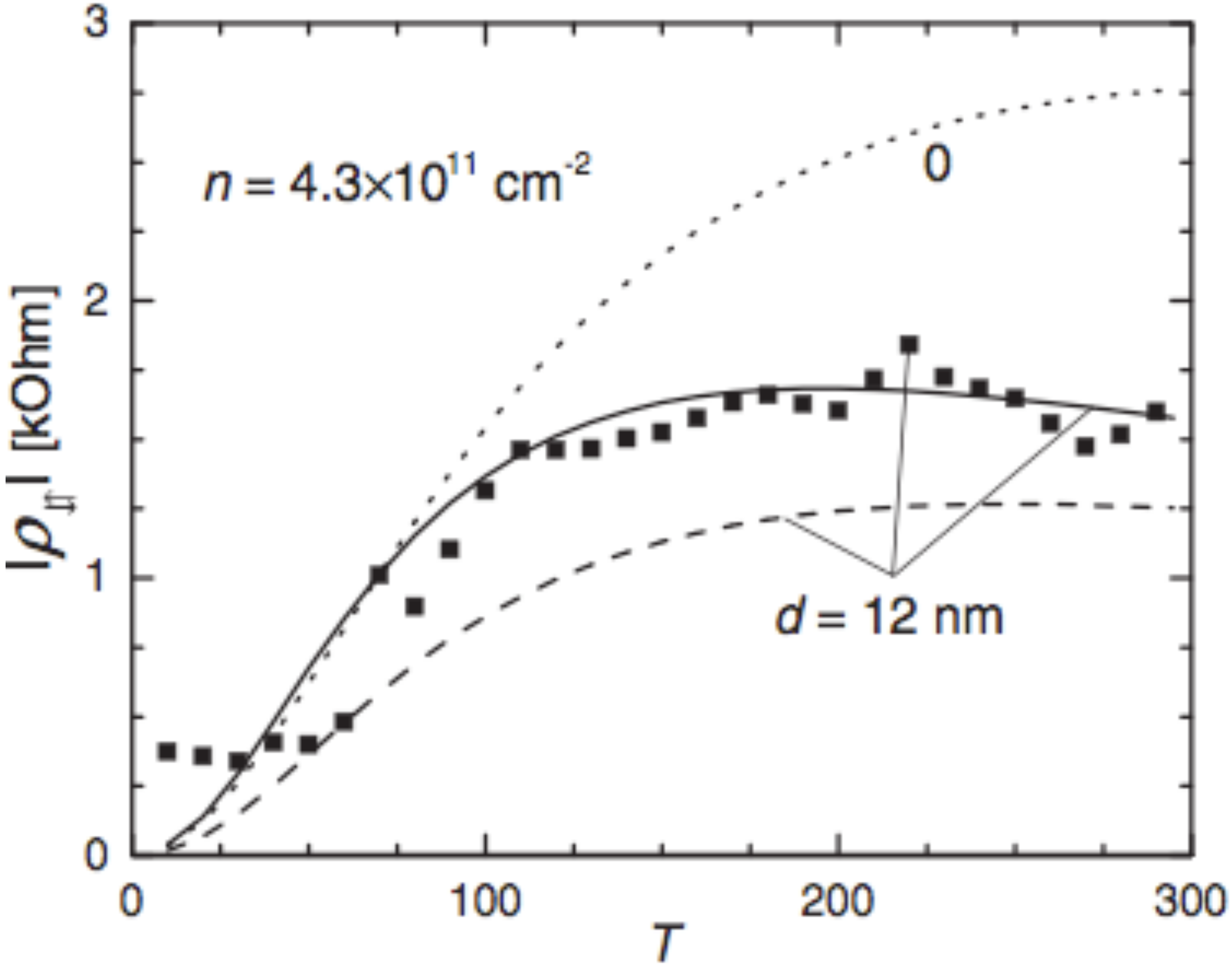
Leading term in the spin drag relaxation rate starts at second-order

$$\frac{1}{\tau_{\rm sd}} = \frac{\hbar^2 n}{n_{\uparrow} n_{\downarrow} m k_{\rm B} T} \int_0^{+\infty} \frac{dq}{2\pi}\, q^2 v_q^2 \int_0^{+\infty} \frac{d\omega}{\pi}\, \frac{\Im m \chi_{\uparrow}^{(0)}(q,\omega) \Im m \chi_{\downarrow}^{(0)}(q,\omega)}{\sinh^2[\hbar\omega/(2k_{\rm B}T)]}$$

I. D'Amico and G. Vignale, Phys. Rev. B **62**, 4853 (2000)

Spin Coulomb drag: experimental



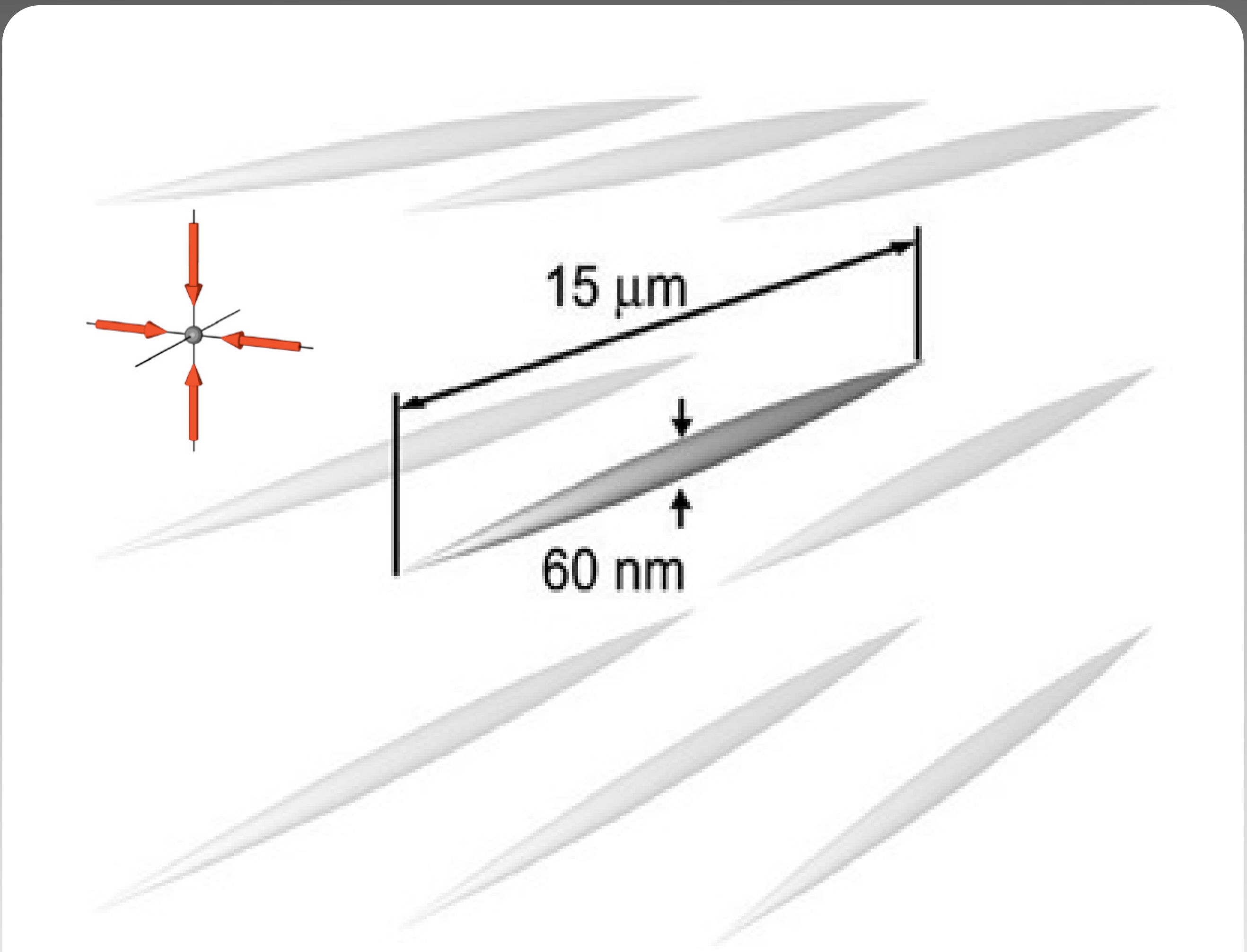


Exp: C.P. Weber et al., Nature **437**, 1330 (2005)
Theory: S.M. Badalyan, C.S. Kim, and G. Vignale, Phys. Rev. Lett. **100**, 016603 (2008)

Spin drag in two-component cold Fermi gases?

MP and G. Vignale, Phys. Rev. Lett. **98**, 266403 (2007)

Spin and charge dynamics in a 1D cold Fermi gas



array of 1D tubes

$$\hat{\mathcal{H}} = \sum_i \frac{\hat{p}_i^2}{2m} + g_{1\mathrm{D}} \sum_{i<j} \delta(\hat{x}_i - \hat{x}_j)$$
$$+ \quad \text{external potential}$$

tunable interaction strength...

$$g_{1\mathrm{D}} = \frac{2\hbar^2 a_{3\mathrm{D}}}{m a_\perp^2} \frac{1}{1 - \mathcal{A} a_{3\mathrm{D}}/a_\perp}$$

M. Olshanii, Phys. Rev. Lett. **81**, 938 (1998)

"Charge":

atomic mass density

"Spin":

two internal (hyperfine) atomic states (e.g. ^{6}Li)

A. Recati, P.O. Fedichev, W. Zwerger, and P. Zoller, Phys. Rev. Lett. **90**, 020401 (2003)

Wavepacket dynamics within linear-response theory

Density-density linear-response function

$$\chi_{\rho\rho}^{-1}(q,\omega)n(q,\omega)=0$$

Spin-spin linear-response function

$$\chi_{S_zS_z}^{-1}(q,\omega)s(q,\omega)=0$$

The problem we want to solve boils down to:

(i) calculating the small q limit of the response functions above

(ii) converting these equations into partial differential equations for **density** and **spin** packets

MP and G. Vignale, Phys. Rev. Lett. **98**, 266403 (2007)

Small q limit of the linear-response functions

Density-density linear-response function

$$\chi^{-1}_{\rho\rho}(q \to 0, \omega) = \frac{m\omega^2}{nq^2} - \frac{m}{n} v_{\rm F}^2 \frac{\kappa_0}{\kappa}$$

Single pole at the sound velocity: $v_\rho = v_{\rm F}\ (\kappa_0/\kappa)^{1/2}$

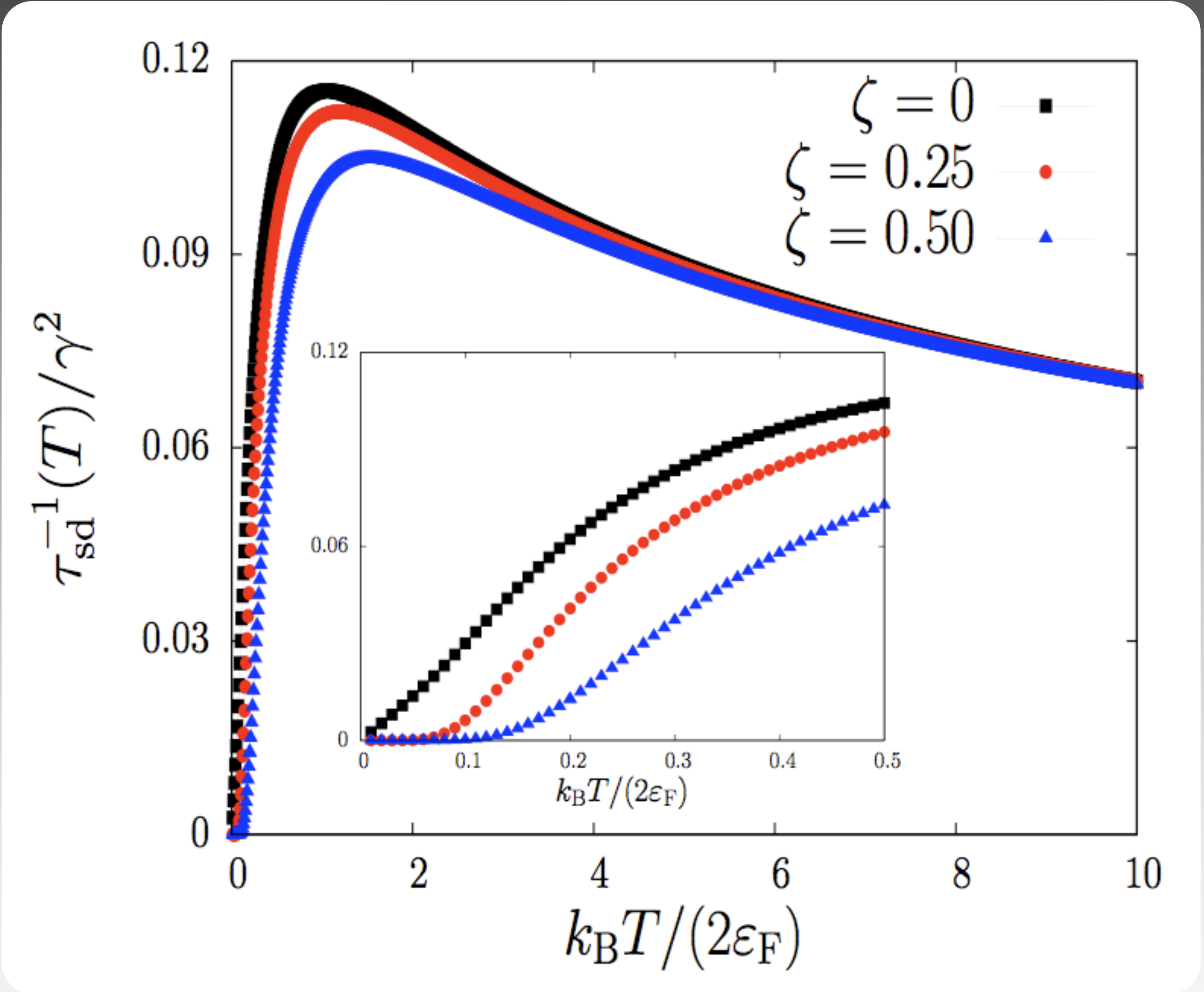
Spin-spin linear-response function

$$\chi^{-1}_{S_z S_z}(q \to 0, \omega) = \frac{m_\sigma \omega(\omega + i\tau_{\rm sd}^{-1})}{nq^2} - \frac{m}{n} v_{\rm F}^2 \frac{\chi_{\sigma 0}}{\chi_\sigma}$$

Single damped pole at the spin velocity: $v_\sigma = v_{\rm F}\ \chi_{\sigma 0}/\chi_\sigma$

MP and G. Vignale, Phys. Rev. Lett. **98**, 266403 (2007)

Spin-drag relaxation rate in 1D



$$\frac{1}{\tau_{\rm sd}(T)} \stackrel{T\to 0}{\longrightarrow} \left[\frac{8}{9\pi}\gamma_{2k_{\rm F}}^2 \frac{k_{\rm B}T}{2\varepsilon_{\rm F}} + \frac{8}{3\pi}\gamma_0^2 \left(\frac{k_{\rm B}T}{2\varepsilon_{\rm F}}\right)^2\right] \frac{\varepsilon_{\rm F}}{\hbar}$$

MP and G. Vignale, Phys. Rev. Lett. **98**, 266403 (2007)

D. Rainis et al., Phys. Rev. B **77**, 035113 (2008)

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Differential equations for density and spin packets

Density channel (a simple D'Alembert equation)

$$\left(v_\rho^{-2}\partial_t^2 - \partial_x^2\right) n(x,t) = 0$$

Spin channel: a damped D'Alembert equation

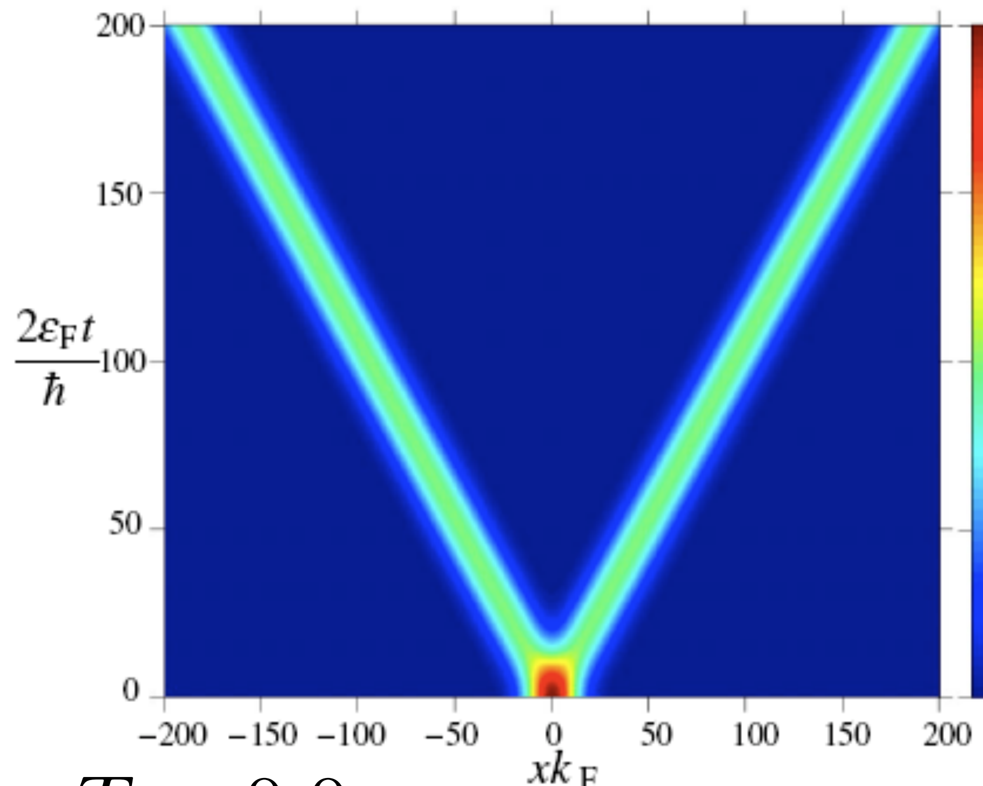
$$\left(v_\sigma^{-2}\partial_t^2 - \partial_x^2\right) s(x,t) + D_\sigma^{-1}\partial_t s(x,t) = 0$$

Spin diffusion constant

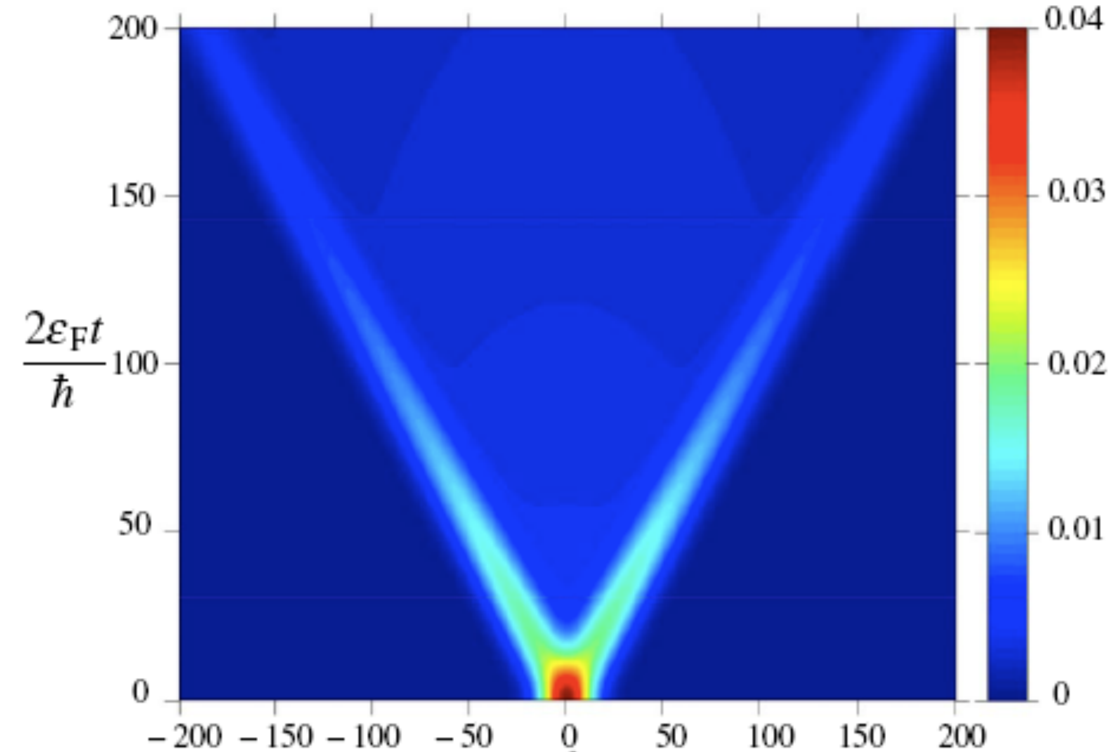
$$D_\sigma = v_\sigma^2 \tau_{\rm sd}(T) = \frac{n\tau_{\rm sd}(T)}{m_\sigma \chi_\sigma}$$

MP and G. Vignale, Phys. Rev. Lett. **98**, 266403 (2007)

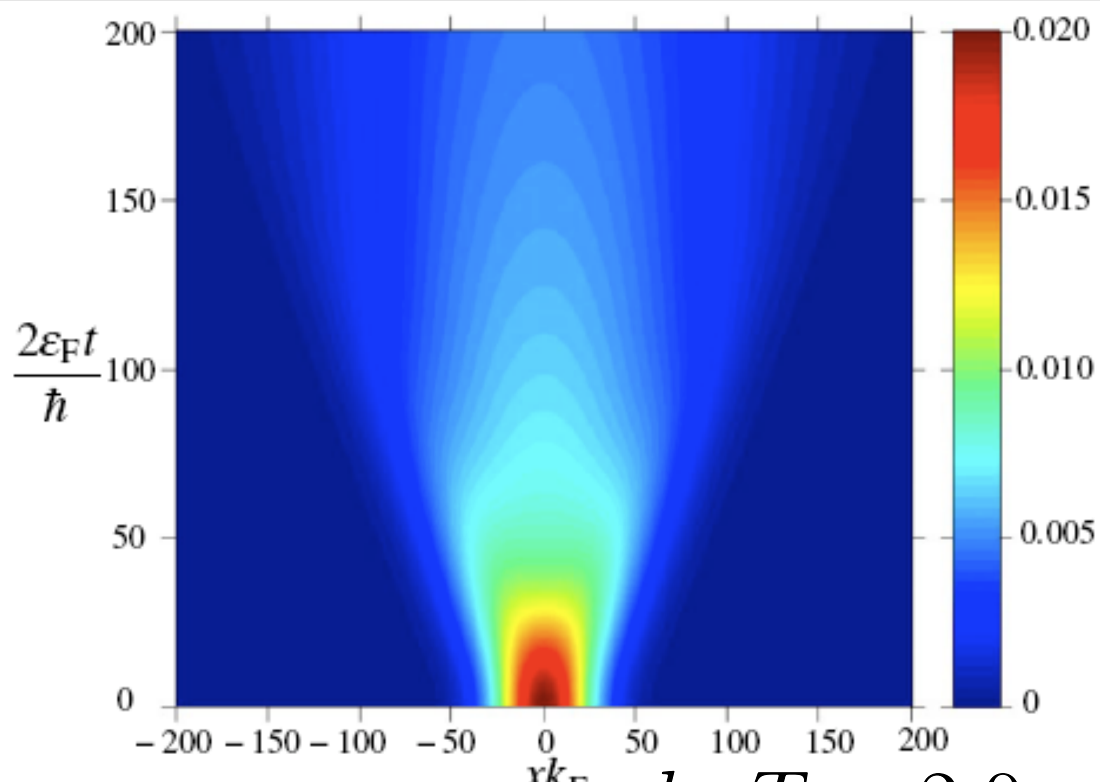
Numerical results



$k_BT = 0.0\ \varepsilon_F$



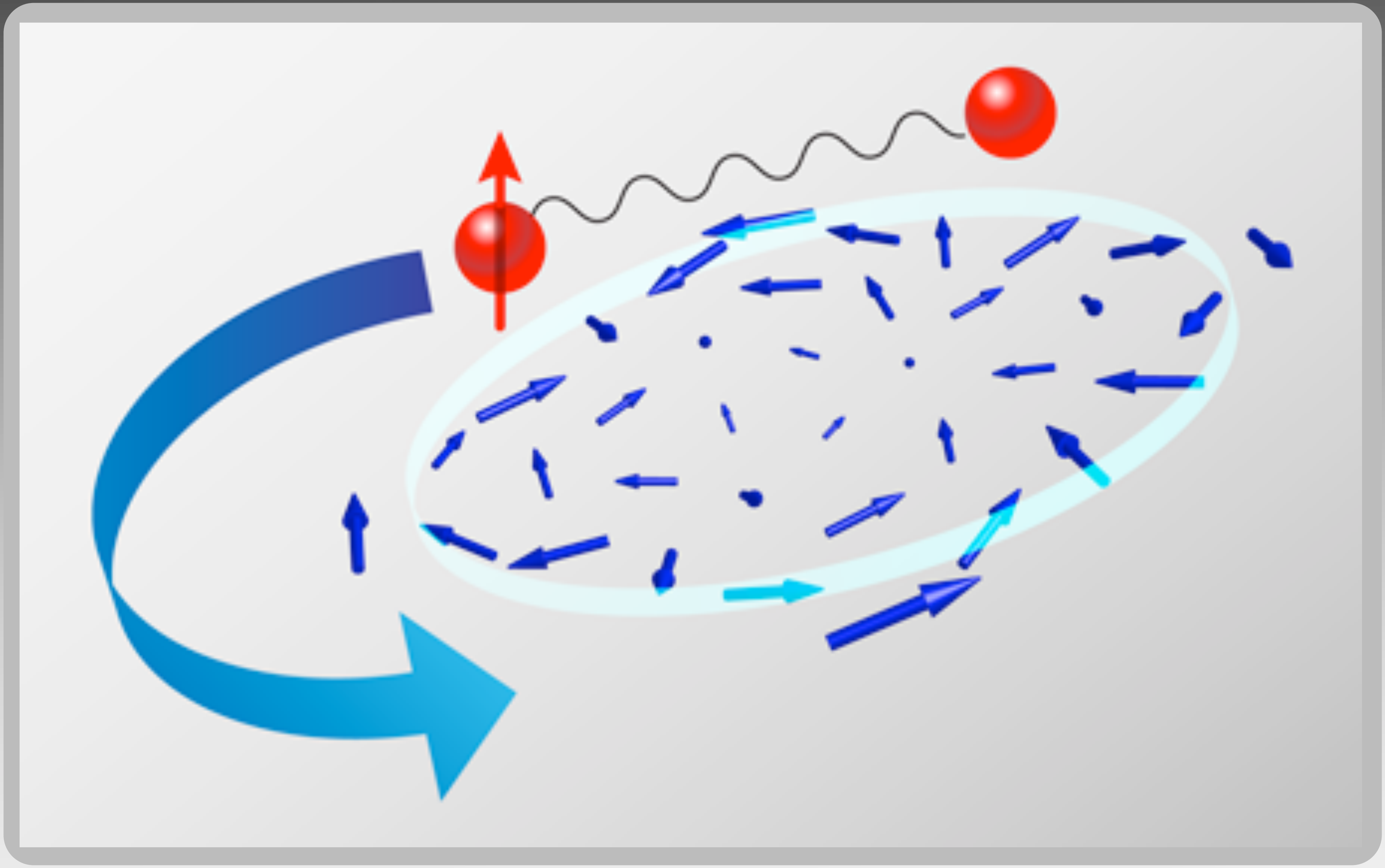
$k_BT = 1.0\ \varepsilon_F$



$k_BT = 2.0\ \varepsilon_F$

MP and G. Vignale, Phys. Rev. Lett. **98**, 266403 (2007)

Spin-drag in a two component Bose gas



$$\frac{1}{\tau_{\rm sd}} \sim T^{-5/2}$$

R.A. Duine and H.T.C. Stoof, Phys. Rev. Lett. **103**, 170401 (2008)

see also related Viewpoint: MP and G. Vignale, Physics **2**, 87 (2009)

Our work motivated by experimental evidence of ferromagnetic correlations in a trapped two-component Fermi gas

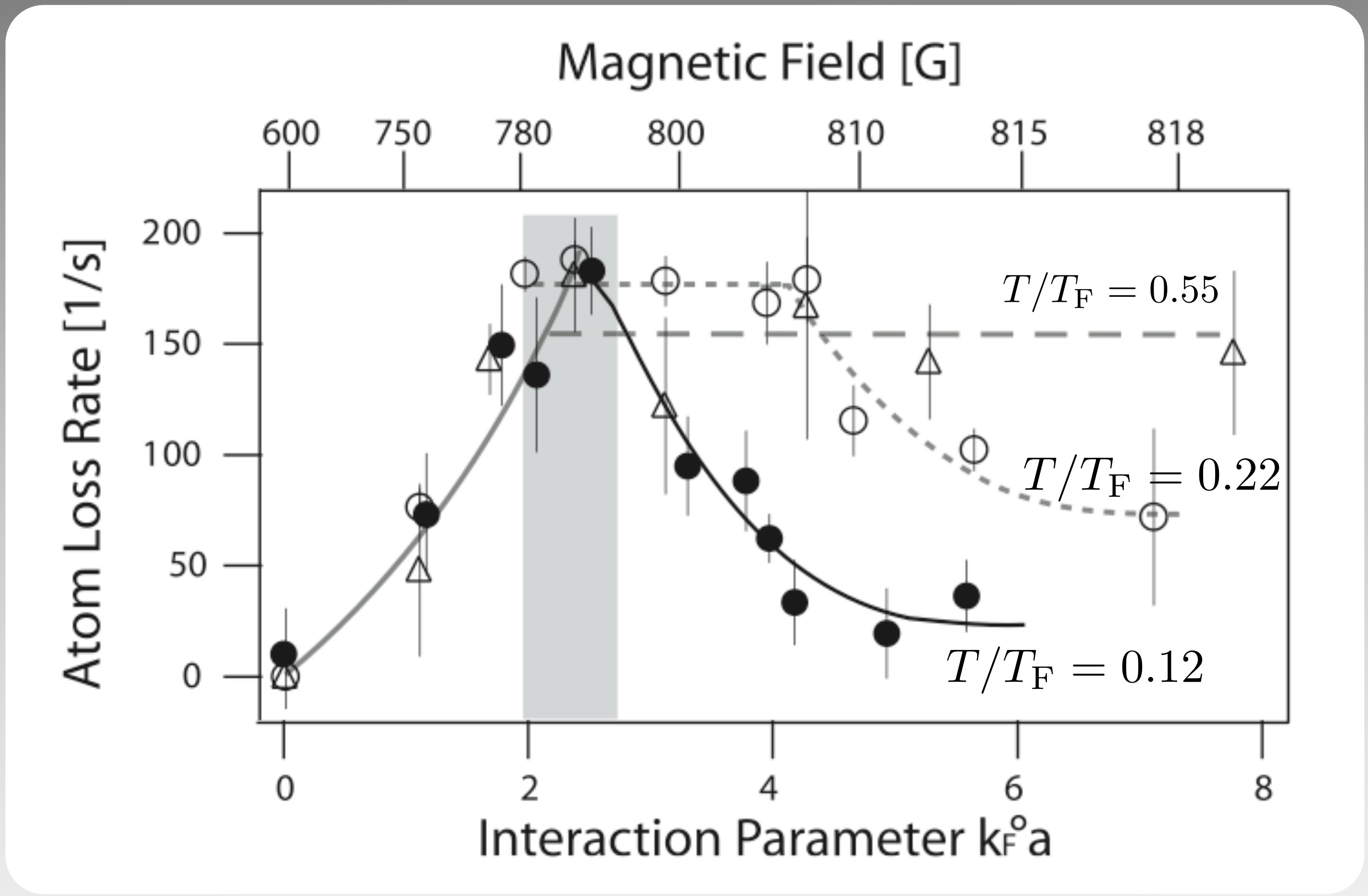
G-B. Jo et al., Science **325**, 1521 (2009)

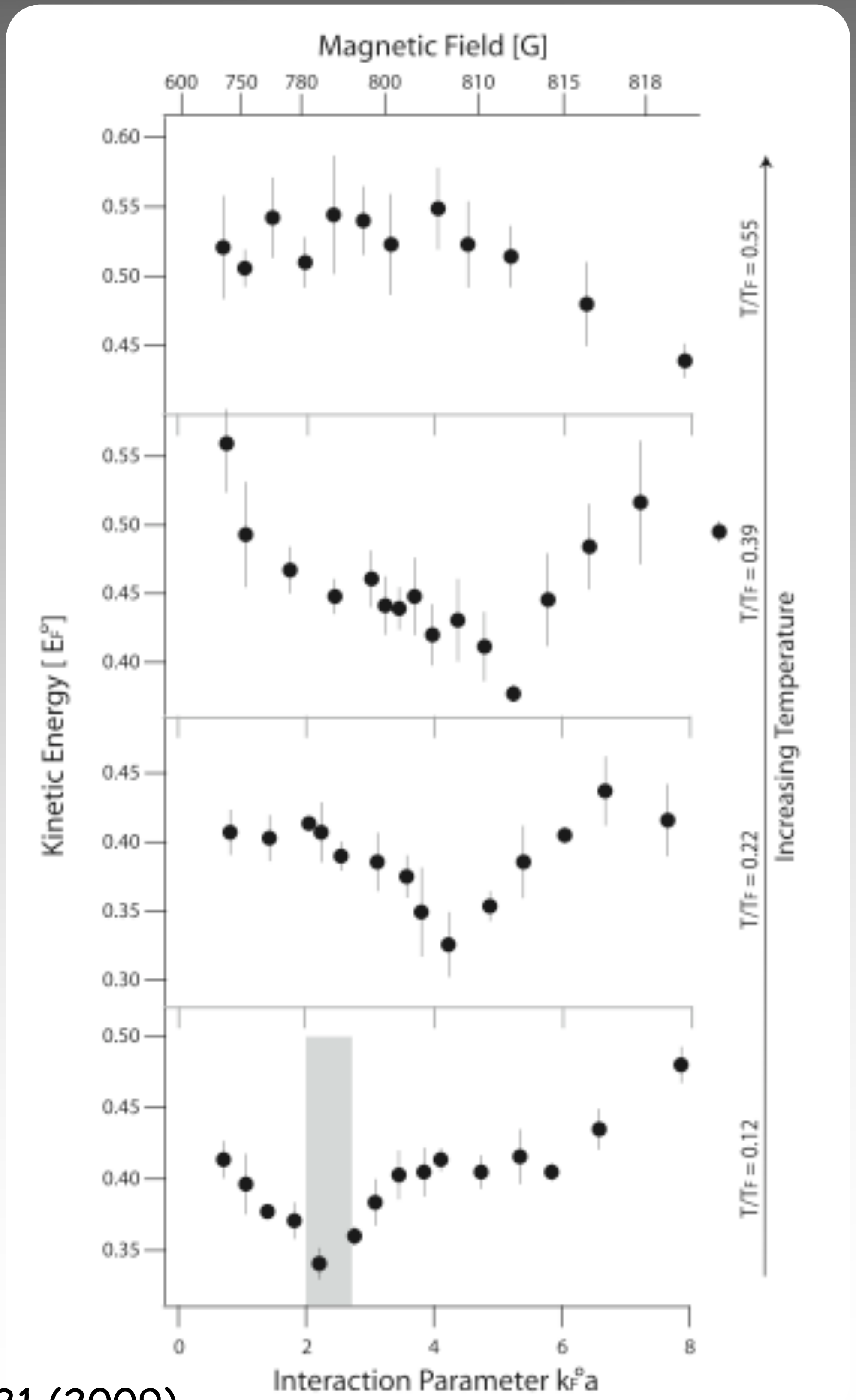
For earlier theoretical work on ferromagnetism see e.g.:
M. Houbiers et al. Phys. Rev. A **56**, 4864 (1997)
L. Salasnich et al., J. Phys. B: At. Mol. Opt. Phys. **33**, 3943 (2000)
M. Amoruso et al., Eur. Phys. J. D **8**, 361 (2000)
T. Sogo and H. Yabu, Phys. Rev. A **66**, 043611 (2002)
R.A. Duine and A.H. MacDonald, Phys. Rev. Lett. **95**, 230403 (2005)

The experiment is not yet well understood by it stimulated a great deal of discussion:

G.J. Conduit and B.D. Simons, Phys. Rev. Lett. **103**, 200403 (2009)
H. Zhai, Phys. Rev. A **80**, 051605(R) (2009)
M. Babadi et al., arXiv:0908.3483v2 ...
... and many others (including recent QMC work)

Itinerant ferromagnetism in a Fermi gas of cold atoms





G-B. Jo et al., Science **325**, 1521 (2009)

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Stoner ferromagnetism (I)

Minimal model: competition between kinetic energy and short-range repulsive interactions between antiparallel-spin fermions:

$$\hat{\mathcal{H}} = \int d^3\boldsymbol{x} \sum_{\alpha\in\{\uparrow,\downarrow\}} \hat{\psi}^\dagger_\alpha(\boldsymbol{x}) \left(-\frac{\hbar^2 \nabla_{\boldsymbol{x}}{}^2}{2m} - \mu\right) \hat{\psi}_\alpha(\boldsymbol{x}) + U \int d^3\boldsymbol{x}\, \hat{\psi}^\dagger_\uparrow(\boldsymbol{x}) \hat{\psi}^\dagger_\downarrow(\boldsymbol{x}) \hat{\psi}_\downarrow(\boldsymbol{x}) \hat{\psi}_\uparrow(\boldsymbol{x})$$

Density-density linear-response function

$$\chi_{nn}(q,\omega) = \frac{\chi_0(q,\omega)}{1 - \frac{U}{2}\chi_0(q,\omega)}$$

Spin-spin linear-response function

$$\chi_{S_z S_z}(q,\omega) = \frac{\chi_0(q,\omega)}{1 + \frac{U}{2}\chi_0(q,\omega)}$$

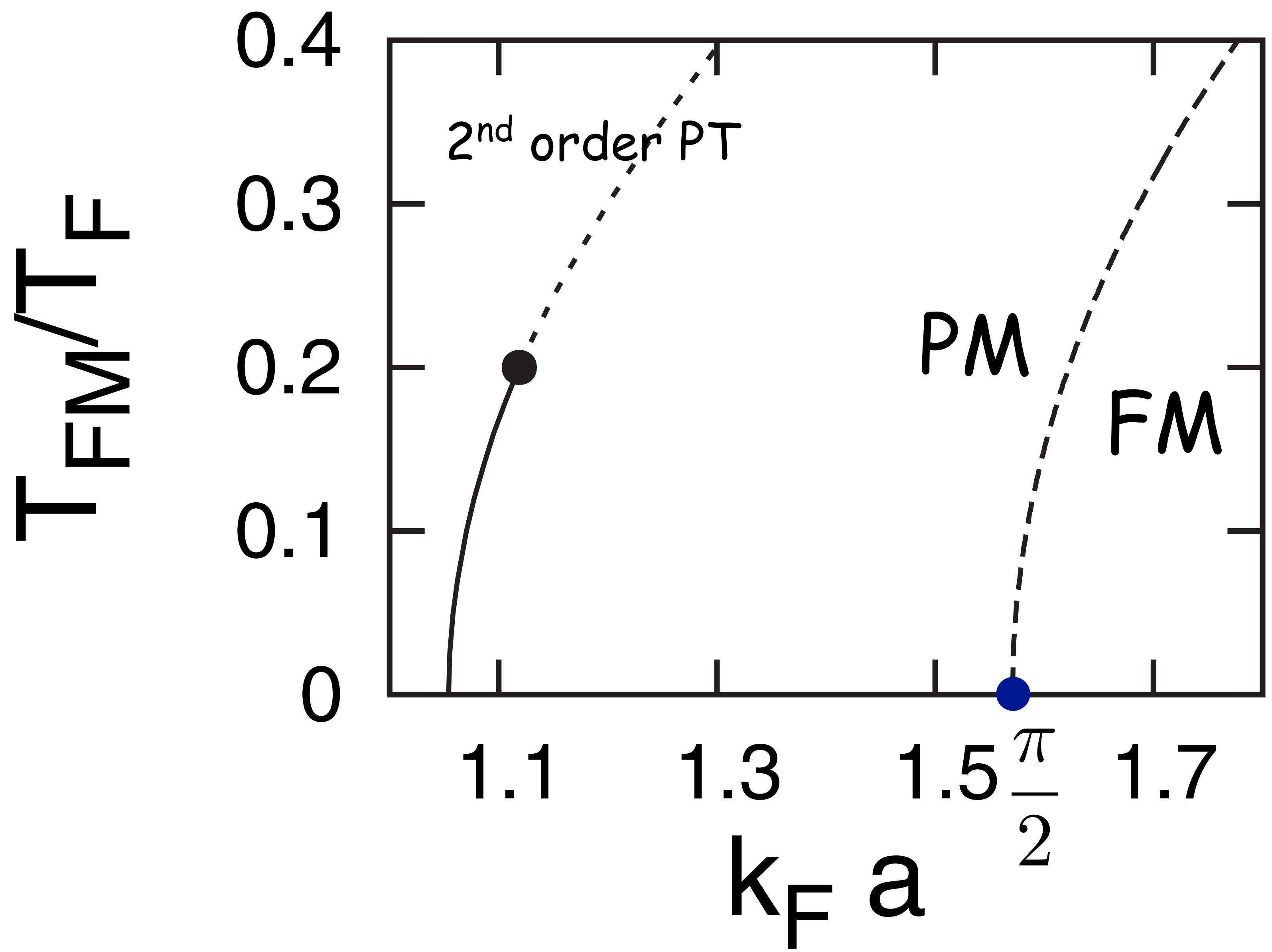
Stoner criterion for ferromagnetism:

$$1 + \frac{U}{2} \lim_{q\to 0} \lim_{\omega\to 0} \chi_0(q,\omega) = 0$$

Stoner ferromagnetism (II)

Stoner criterion for ferromagnetism:

$$1 + \frac{U}{2} \lim_{q \to 0} \lim_{\omega \to 0} \chi_0(q, \omega) = 0$$



R.A. Duine and A.H. MacDonald, Phys. Rev. Lett. **95**, 230403 (2005)

Spin-drag relaxation rate

Boltzmann transport and collision integral

Scattering amplitude

$$I_{\rm coll}[f_{\boldsymbol{k},\uparrow}] \propto \int \frac{d^D \boldsymbol{k}'}{(2\pi)^D} \int \frac{d^D \boldsymbol{q}}{(2\pi)^D} \int_{-\infty}^{+\infty} d\omega\ |A_{\uparrow\downarrow}(q,\omega)|^2 [f_{\boldsymbol{k},\uparrow}(1-f_{\boldsymbol{k}+\boldsymbol{q},\uparrow}) f_{\boldsymbol{k}',\downarrow}(1-f_{\boldsymbol{k}'-\boldsymbol{q},\downarrow}) - f_{\boldsymbol{k}+\boldsymbol{q},\uparrow}(1-f_{\boldsymbol{k},\uparrow}) f_{\boldsymbol{k}'-\boldsymbol{q},\downarrow}(1-f_{\boldsymbol{k}',\downarrow})] \delta(\omega - \varepsilon_{\boldsymbol{k}+\boldsymbol{q},\uparrow} + \varepsilon_{\boldsymbol{k},\uparrow}) \delta(\omega + \varepsilon_{\boldsymbol{k}'-\boldsymbol{q},\downarrow} - \varepsilon_{\boldsymbol{k}',\downarrow})$$

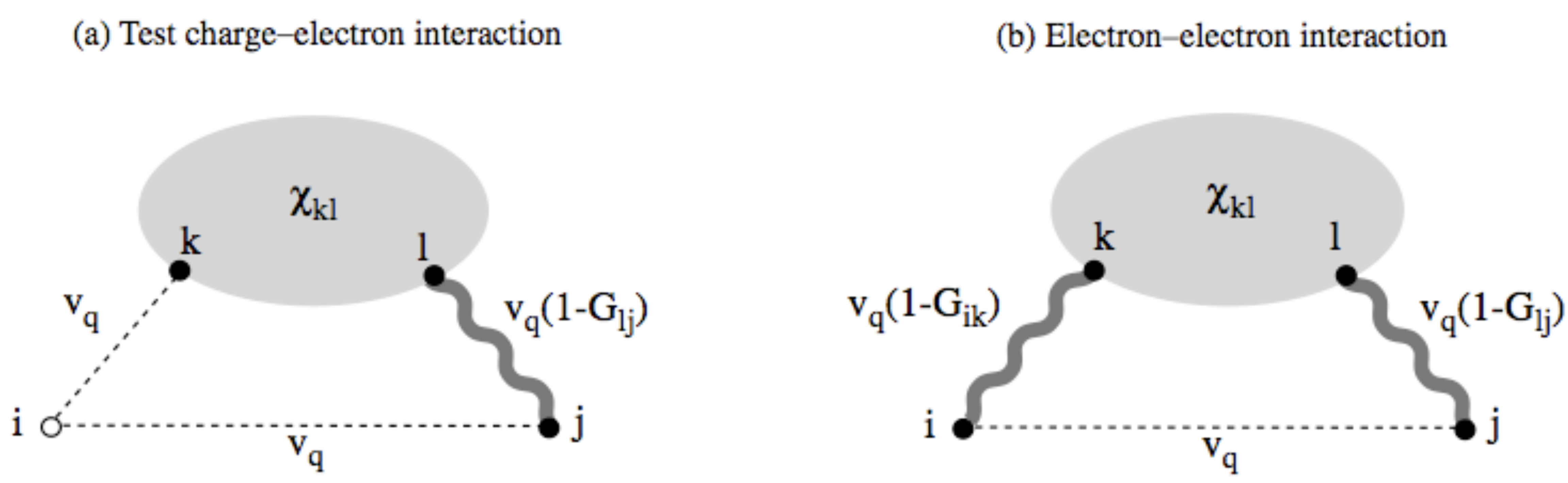
Rate of change of spin-up momentum

$$\frac{d\boldsymbol{P}_\uparrow}{dt} = \sum_{\boldsymbol{k}} \boldsymbol{k}\ I_{\rm coll}[f_{\boldsymbol{k},\uparrow}]$$

Spin-drag relaxation rate above critical temperature

$$\frac{1}{\tau_{\rm sd}(T)} = \frac{1}{4Mnk_{\rm B}T} \int \frac{d^D \boldsymbol{q}}{(2\pi)^D} \frac{q^2}{D} \int_{-\infty}^{+\infty} \frac{d\omega}{\pi} |A_{\uparrow\downarrow}(q,\omega)|^2 \frac{[\Im m\ \chi^{(0)}(q,\omega)]^2}{\sinh^2[\omega/(2k_{\rm B}T)]}$$

Effective interactions



Scattering amplitude: density, longitudinal and transverse spin fluctuations

$$A_{\uparrow\downarrow}(q,\omega) = \underbrace{U}_{\text{direct term}} + \underbrace{\frac{U^2}{4}\,\chi_{\rho\rho}(q,\omega)}_{\text{density fluctuations}} - \underbrace{\frac{U^2}{4}\,\chi_{S_zS_z}(q,\omega)}_{\parallel\text{ spin fluctuations}} - \underbrace{2\frac{U^2}{4}\,\chi_{S_zS_z}(q,\omega)}_{\perp\text{ spin fluctuations}}$$

C.A. Kukkonen and A.W. Overhauser, Phys. Rev. B **20**, 550 (1979)
G.F. Giuliani and G. Vignale, Quantum Theory of the Electron Liquid (CUP, Cambridge, 2005)
see also A.V. Chubukov and D.L. Maslov, Phys. Rev. Lett. **103**, 216401 (2009)

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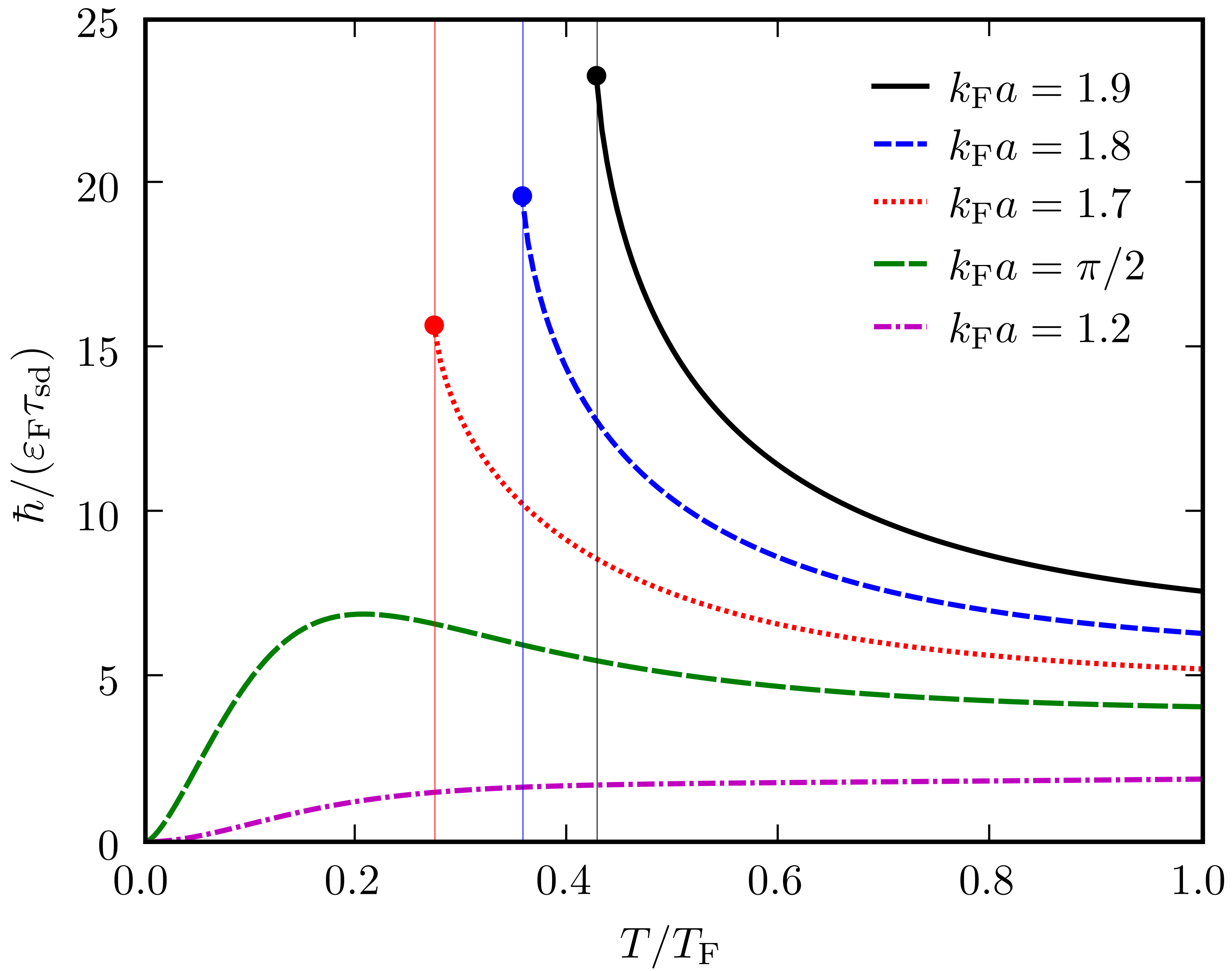
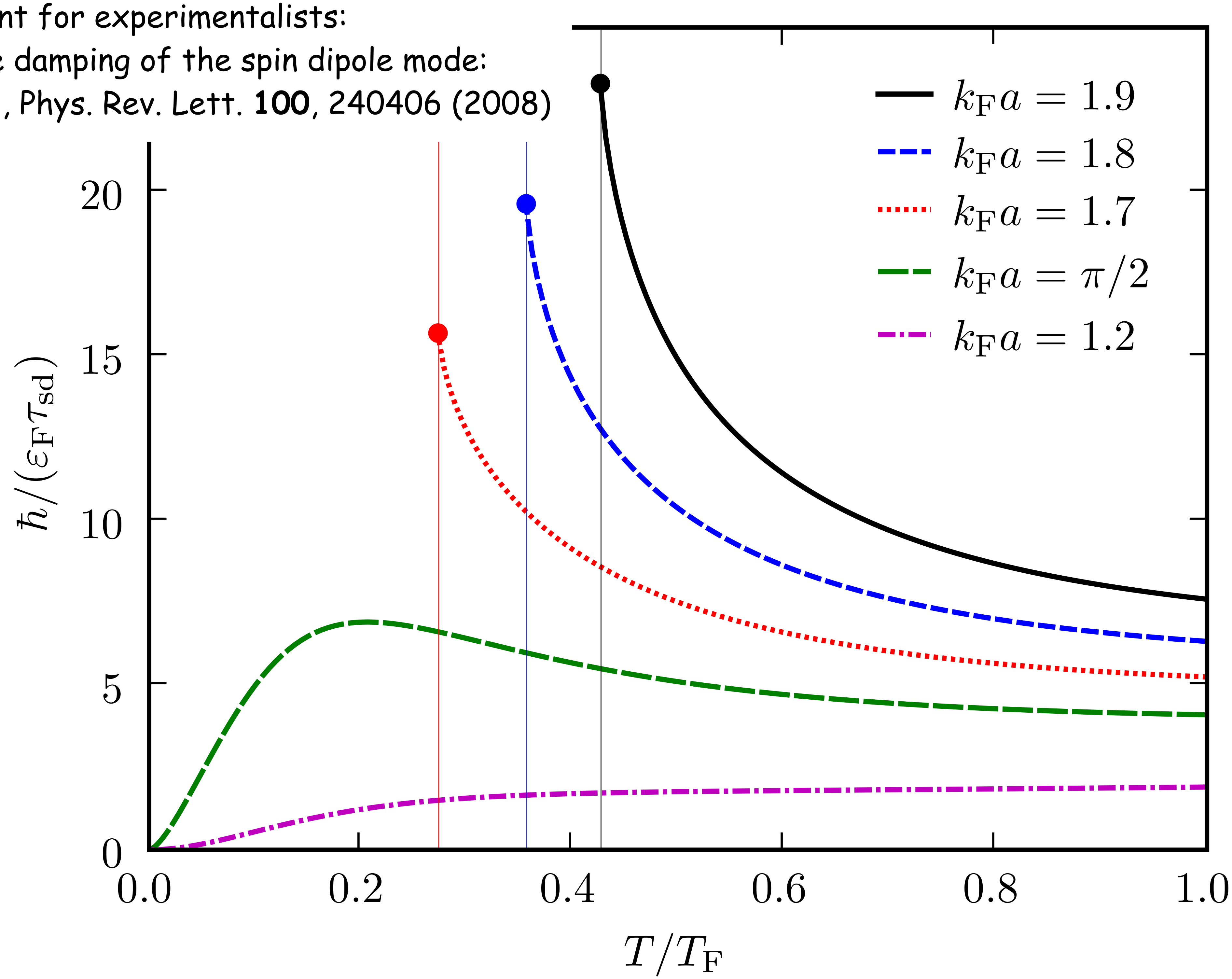
Temperature dependence of the spin-drag relaxation rate



R.A. Duine, MP, H.T.C. Stoof, and G. Vignale, Phys. Rev. Lett. **104**, 220403 (2010)

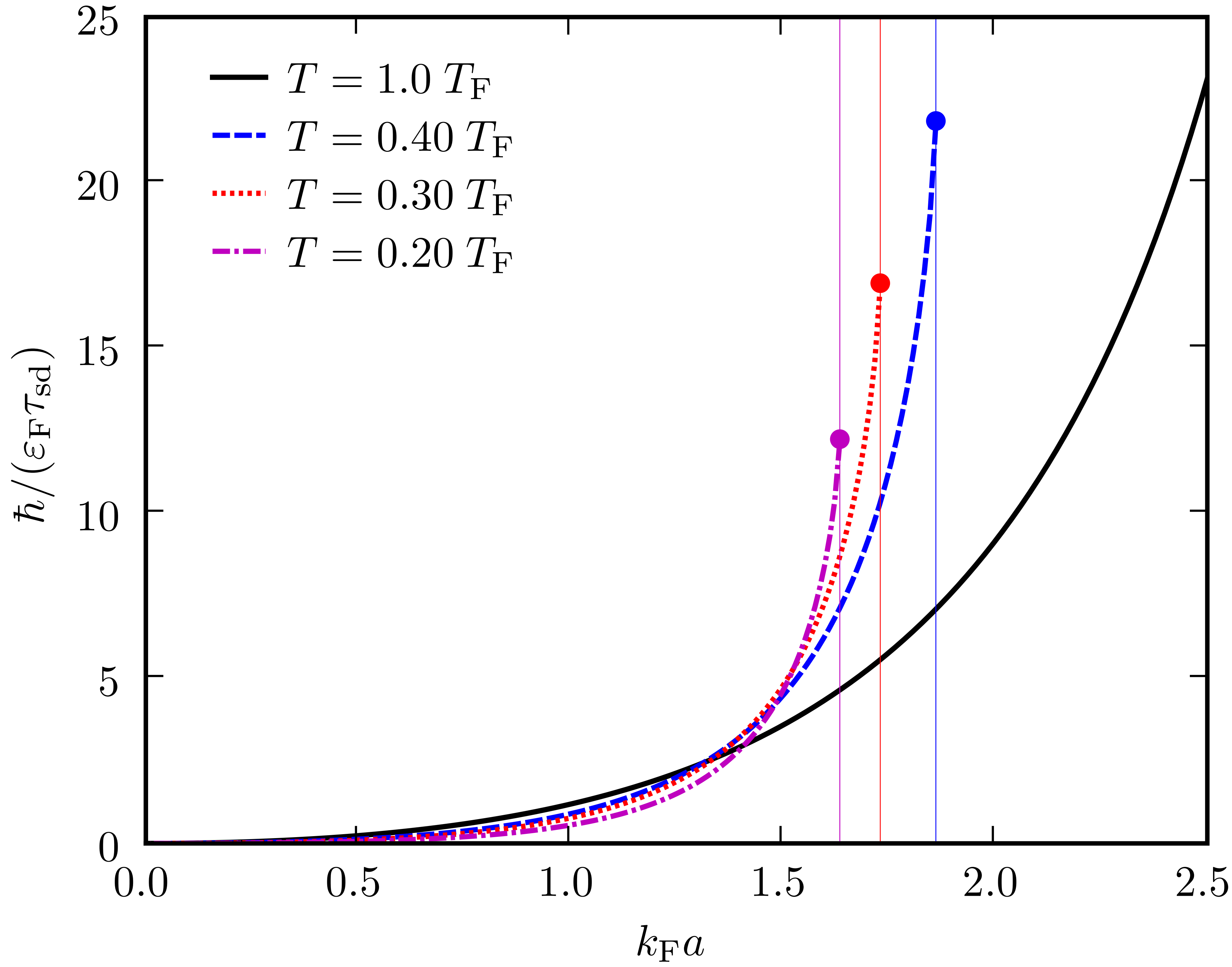
Temperature dependence of the spin-drag relaxation rate

Hint for experimentalists:
measure the damping of the spin dipole mode:
G.M. Bruun et al., Phys. Rev. Lett. **100**, 240406 (2008)



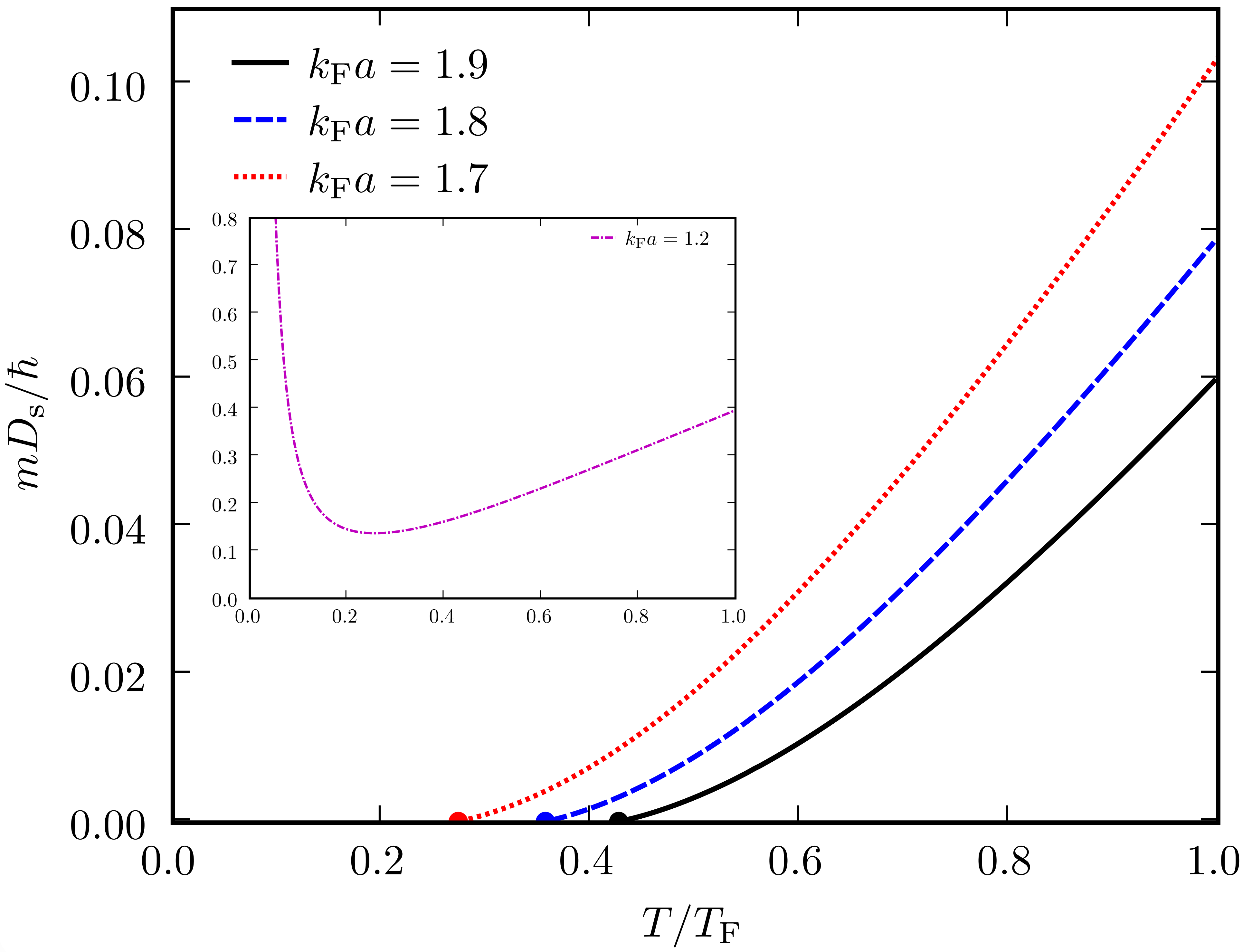
R.A. Duine, MP, H.T.C. Stoof, and G. Vignale, Phys. Rev. Lett. **104**, 220403 (2010)

The spin-drag relaxation rate as a function of interaction strength



R.A. Duine, MP, H.T.C. Stoof, and G. Vignale, Phys. Rev. Lett. **104**, 220403 (2010)

Temperature dependence of the spin diffusion constant



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- We have shown that when the ferromagnetic state is approached from the normal side, the spin-drag relaxation rate is strongly enhanced near the critical point

- We have also determined the temperature dependence of the spin diffusion constant

- In a trapped gas, the spin-drag relaxation rate determines the damping of the spin dipole mode, which therefore provides a precursor signal of the ferromagnetic phase transition that may be used to experimentally determine the proximity to the ferromagnetic phase

- What's next? Currently extending the theory to $T < T_c$, to lower dimensionality, and to electron-hole bilayers

Thank you for your attention!

For more details please take a look at:

R.A. Duine, MP, H.T.C. Stoof, and G. Vignale, Phys. Rev. Lett. **104**, 220403 (2010)

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