Extended hydrodynamics from the BBGKY hierarchy

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It is known that the set of densities of conserved quantities forms the basis for description of a simple fluid at the level of usual hydrodynamics, when the slowest hydrodynamical processes are considered. Balance equations for momentum and energy densities become closed due to the Navier-Stokes and Fourier laws for the stress tensor and heat flux together with expressions for transport coefficients.

The idea of extension of the hydrodynamic description consists in treating the momentum and energy fluxes as independent variables which supplement and extend the set of densities of the conserved quantities. The corresponding transport equations for these fluxes are to be introduced. For the first time, this was done by Grad for rarefied gases in the well-known moment method for solving the Boltzmann kinetic equation [1]. In recent papers [2–4], the Grad moment method has been applied to the Enskog kinetic equation for hard sphere system.

In this connection, we propose general approach to derivation of exact balance equations in terms of fluxes and sources for arbitrary hydrodynamic-like densities based on the use of the BBGKY hierarchy for the cases of a smooth continuous potential and a hard-sphere repulsion. For many-particle local and two-particle nonlocal hydrodynamic densities, the corresponding balance equations valid for any number density are obtained and explicit expressions for contributions of different types to their fluxes and sources are deduced.

As the first step, the scheme is applied to the momentum and energy fluxes so that balance equations for them make the first extention of the usual hydrodynamics. The counterpart equations for the stress tensor and the heat flux related to the local reference frame are found out. It is anticipated that the next extensions develop out in an infinite hierarchy of hydrodynamic-type transport equations. Comparison of the obtained results for the two mentioned types of interaction is provided. Absent in the smooth potential case, some peculiarities for variables of the first and higher orders are noticed.

It is expected that the extended set of balance equations, supplied with proper closure relations, can describe faster hydrodynamic processes of relaxation type as it was shown by Grad for the Boltzmann gases.

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