Instability of the perturbative RG flow and critical exponents

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The perturbative renormalization of the Ginzburg–Landau model is reconsidered based on the Feynman diagram technique. We derive closed equations including exactly all vertices appearing in the perturbative renormalization of the φ^4 model up to the order ε^3 of the ε-expansion. The renormalized Hamiltonian contains terms corresponding to different φ^2, φ^4, φ^6, and φ^8 vertices. All these terms are relevant. We have tested the expected basic properties of the renormalization group (RG) flow, such as the semigroup property $R_{s_1 s_2}\mu = R_{s_2} R_{s_1}\mu$, where R_s is the RG operator with scale factor $s > 1$ acting on the set of parameters μ. Besides, the existence of the fixed point and its independence of the parameter s have been verified. All these properties are satisfied, if the RG flow equations are truncated at the order of ε^2. However, our analysis reviels a problem in the next order of the ε–expansion, i.e., the fixed point is unstable in this case even for the critical parameters of the model. We have tested also a modified approach, where the φ^4 coupling constant u is the expansion parameter at a fixed spatial dimensionality d. In addition to the instability problem, our tests point to an internal inconsistency of such a method.

The observed instability of the perturbative RG flow allows the following interpretation: the perturbative RG theory describes a transient behaviour rather than the true critical behaviour of the Ginzburg–Landau model. This scenario is supported by the values of the critical exponents, which are exact in view of alternative theoretical treatments [1,2], being inconsistent with those of the perturbative RG theory. It is supported also by an experimental evidence [3] and our recent Monte Carlo simulations of the 3D Ising model with linear lattice sizes up to $L = 1024$.

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