Dimensional reduction of diffusion in confined systems

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We consider bulk diffusion in a narrow channel of varying cross section $A(x)$. We present a rigorous recurrence procedure of mapping the 2D (3D) diffusional equation onto the longitudinal direction x. The result is a generalized Fick-Jacobs equation. This equation is compared with a similar equation known from the nonequilibrium thermodynamics, which modifies the flux by an effective diffusion coefficient $D(x)$. We show that the exact mapping meets the phenomenological description in the limit of stationary flow. Then the function $D(x)$ can be expressed by using our mapping procedure. $D(x)$ appears to be related to the stationary density in the channel and so there exist other methods how to calculate it. We demonstrate some of them, providing approximate formulas for $D(x)$, which are applicable in practice.

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